Elementary Mathematics Framework
# Table of Contents

**Section 1: FHSD Philosophy & Policies**
- FHSD Mathematics Course of Study (board documents) ........ pages 2-3
- FHSD Technology Statement.................................................. page 4
- FHSD Mathematics Calculator Policy........................................ pages 5-7

**Section 2: FHSD Mathematical Practices**
- Description.................................................................................. page 8
- Instructional Guidance by grade level band ......................... pages 9-32
- Look-for Tool................................................................................. pages 33-35
- Mathematical Practices Classroom Visuals............................... pages 36-37

**Section 3: FHSD Mathematical Teaching Habits (NCTM)**
- Description.................................................................................. page 38
- Mathematical Teaching Habits.................................................... pages 39-54

**Section 4: RtI: Response to Intervention**
- RtI: Response to Intervention Guidelines................................. pages 55-56
- Skills and Scaffolds................................................................. page 56
- Gifted............................................................................................... page 56
Section 1: Philosophy and Policies

Math Course of Study, Board Document 2015

Introduction
A team of professional, dedicated and knowledgeable K-12 district educators in the Forest Hills School District developed the Math Course of Study. This document was based on current research in mathematics content, learning theory and instructional practices. The Ohio’s New Learning Standards and Principles to Actions: Ensuring Mathematical Success for All were the main resources used to guide the development and content of this document.

While the Ohio Department of Education’s Academic Content Standards for School Mathematics was the main source of content, additional sources were used to guide the development of course indicators and objectives, including the College Board (AP Courses), Achieve, Inc. American Diploma Project (ADP), the Ohio Board of Regents Transfer Assurance Guarantee (TAG) criteria, and the Ohio Department of Education Program Models for School Mathematics.

The Mathematics Course of Study is based on academic content standards that form an overarching theme for mathematics study. Six standards flow through mathematics instruction, preschool through high school. The first five are: 1) Number, Number Sense and Operations, 2) Measurement, 3) Geometry and Spatial Sense, 4) Patterns, Functions and Algebra, and 5) Data Analysis and Probability. The sixth Standard is a process standard and includes: 1) Problem Solving, 2) Communication, 3) Connections, 4) Reasoning, and 5) Representation.

The content in the Forest Hills School District Mathematics Course of Study reflects the skills, processes and knowledge we believe students in the Forest Hills School District need to know to be competent, knowledgeable and confident in their understanding of mathematics and in their ability to apply this understanding in future learning experiences.

Philosophy
Learning mathematics involves a variety of skills, processes and understandings. We support a balanced approach to teaching mathematics, which emphasizes both conceptual understanding and procedural fluency and which leads to increased mathematical proficiency for all students in Forest Hills. We believe students must have a comprehension of mathematical concepts, operations and relations as well as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. This is best accomplished through a well-articulated, comprehensive, coherent and consistent mathematics curriculum in all Forest Hills schools, from early childhood years through high school graduation.

Along with conceptual understanding and procedural fluency, students must also have an ability to formulate, represent and solve a variety of mathematical problems. Flexibility, creative thinking, and strategic competence are essential in today’s information-driven, quickly changing world. We strive to educate students who not only have the understanding of mathematical concepts and procedural fluency with mathematical skills but who can model mathematical situations appropriately and construct their own learning through carefully planned classroom experiences.
An ability to reason is essential, with a capacity for logical thought, reflection, explanation and justification. Communicating mathematically verbally and in written form is an important step in leading to greater mathematical proficiency, and all students should frequently be asked to reason, communicate, make connections, and represent mathematical ideas.

We as educators will aspire to help all students learn to appreciate mathematics and see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own experience. Research confirms that parent involvement is an important factor in student achievement. We encourage communication with parents about our mathematics teaching and learning and we value their support of the mathematics.

None of the components mentioned above is independent, and all are interwoven and interdependent in the development of mathematical proficiency. The classroom teacher of mathematics is the most important factor in weaving the strands together through daily instruction, differentiating to meet the needs of all students, and collaborating with colleagues to help all students achieve success in mathematics learning.
Technology Statement
As technology becomes essential in our daily activities, the world demands more technological expertise. The vast array of technologies available to teachers should be integrated as a modern tool of instruction to make mathematics more meaningful and accessible for all students.

Forest Hills Calculator Belief Statements and Decision Making Framework:
Calculators should be used when they enhance student learning.

Calculators should be used when the computations become so cumbersome that they are an obstacle to learning higher-level concepts.

Students should be taught how to use calculators in responsible and reasonable ways. Most importantly, students should be able to accurately interpret and understand the answers provided by a calculator.

Students should be taught when to use a calculator and when mental computing is more effective or appropriate. Choosing the right “tool” is part of an effective problem-solving process.

Calculators are an essential tool:
- for the discovery of generalizations or patterns within mathematics
- when making sense of mathematics
- for communicating mathematical thinking

Forest Hills calculator belief statements are based upon:
- The Ohio Department of Education Assessment Guidelines
- Research highlighted in Principles to Actions: Ensuring Mathematical Success for All, p. 78-83:
  "Despite popular belief, use of technology does not inhibit students’ learning of mathematics. The idea that it does is particularly prevalent regarding the use of calculators. However, after conducting a comprehensive literature review, Ronau and others (2011, p.1) concluded the following:

  In general, we found that the body of research consistently shows that the use of calculators in the teaching and learning of mathematics does not contribute to any negative outcomes for skill development or procedural proficiency, but instead enhances the understanding of mathematics concepts and student orientation toward mathematics."

- Standards for Mathematical Practice #5:
  Use appropriate tools strategically.
  Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
The following charts were developed by the West Virginia Department of Education for instructional decision making regarding the use of calculators. The Forest Hills Course of Study Committee endorses their use in lesson design, instructional planning, and assessment.

**Calculator Usage in the Primary Grades**

Is calculator being used by teacher or students?

Teacher — Use an overhead or document camera to model the keystrokes for students to follow in a problem solving situation

Students — What is the purpose of the activity?

Explore counting and number patterns or solve problems beyond counting range

Can students perform the needed operations?

Yes — Calculator optional, recommended for checking work

No — Calculator suggested

Developing concepts of counting and meaning of number

No Calculator
Calculator Usage in the Intermediate Grades

What is the focus of the lesson?

Learning Procedures with Whole Numbers, Fractions, Decimals, and Percents
Do Not Use Calculator

Computation with Very Large or Messy Numbers*
Use Calculator

Solving Application Problems in Real-World Contexts?

Is computation the focus?

Yes
Do Not Use Calculator

No
Calculator Optional

*Students with special needs may require modifications including using the calculator when performing multi-digit computation
Operations with Rational Numbers

1. Does student background include the development of conceptual understandings of operations with rational numbers?
   - Yes
     - Engage students in hands-on and pencil and paper investigations to develop conceptual understandings.
   - No
     - Does the learning task involve computation with tedious rational numbers?
       - Yes
         - Can estimation or mental math be used?
           - No
             - Use pencil and paper computation
           - Yes
             - Use estimation or mental math
       - No
         - Use calculator

2. Assess student understanding
Section 2: FHSD Mathematical Practices

The Standards for Mathematical Practices describe ways student practitioners are developed as they engage with content and develop a balanced combination of procedures, processes, and understanding.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

On the following pages you will find a snapshot of each practice by grade band. It includes a detailed definition, kid friendly “I can” statements, examples of teacher’s role and student’s role to guide a teacher’s instructional planning and integrate with the content standards. We use these practices to teach Ohio’s New Learning Standards.

- Grades K-1 Math Practices
- Grades 2-3 Math Practices
- Grades 4-6 Math Practices

The Forest Hills School District Mathematical Practices Look-For Tool was created through a K-12 collaborative team while analyzing the practices both in the classroom and using current research. The tool was designed to be used by teachers to grow professionally through reflection of their practices. An administrator or colleague can provide a teacher feedback using this tool constructively.
**Grades K-1 Math Practices**

**MP1 Makes sense of problems and perseveres in solving them**

<table>
<thead>
<tr>
<th>Definition</th>
<th>Student Friendly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students understand and look for a way to solve the problem. They can explain their process, monitor their work, and prove their solutions.</td>
<td>● I can explain/understand what the problem is asking.</td>
</tr>
<tr>
<td>“Do I understand the problem?”</td>
<td>● I can make a plan.</td>
</tr>
<tr>
<td>“Did I persevere?”</td>
<td>● I can get unstuck when I am stuck.</td>
</tr>
<tr>
<td>“Does my solution make sense?”</td>
<td>● I can change my plan if it isn’t working.</td>
</tr>
<tr>
<td></td>
<td>● I monitor my work.</td>
</tr>
<tr>
<td></td>
<td>● I can prove that my answer makes sense.</td>
</tr>
<tr>
<td></td>
<td>● I can clearly explain my answers.</td>
</tr>
</tbody>
</table>

**Examples:**

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>● provide open-ended rich problems</td>
<td>● share and discuss strategies</td>
</tr>
<tr>
<td>● ask probing questions</td>
<td>● kids work in groups collaboratively</td>
</tr>
<tr>
<td>● model solving multi-step problems</td>
<td>● persevere through frustrations</td>
</tr>
<tr>
<td>● scaffold instruction</td>
<td>● use a variety of strategies and methods to solve a problem</td>
</tr>
<tr>
<td>● provide a safe environment for learning from mistakes and collaborating together</td>
<td>● monitor their problem solving process</td>
</tr>
<tr>
<td>● model how to find key information</td>
<td>● prove their solutions in more than one way</td>
</tr>
<tr>
<td>● model a variety of strategies</td>
<td>● re-read and retell the problem in their own words</td>
</tr>
<tr>
<td>● allow time for student-led discussions</td>
<td>● find key information</td>
</tr>
<tr>
<td>● probe for a variety of different strategies</td>
<td>● check to see if their answer is reasonable</td>
</tr>
</tbody>
</table>

**Non-examples**

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>● teach isolated skills not connected to other learning</td>
<td>● do not explain solutions or problem solving process</td>
</tr>
<tr>
<td>● do not access or build on prior knowledge</td>
<td>● quit due to frustration</td>
</tr>
<tr>
<td>● do not create a safe learning environment for kids to make mistakes.</td>
<td>● do not make connections or use their prior knowledge and experiences</td>
</tr>
<tr>
<td>● ask closed questions versus open-ended</td>
<td>● do not evaluate if their solution is a reasonable answer</td>
</tr>
</tbody>
</table>
# Grades K-1 Math Practices

## MP2 Reason abstractly and quantitatively

<table>
<thead>
<tr>
<th>Definition</th>
<th>Student Friendly</th>
</tr>
</thead>
</table>
| Students can visualize problems. They can share their thinking with others to solve problems and explain their solution. Students can create multiple representations to explain their thinking (using pictures, words, number sentences, tables, graphs). | - I can make sense of the problem.  
- I can use numbers, pictures, words, equations to show my thinking.  
- I can prove that my answer makes sense. |

### Examples

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| - develop opportunities for problem solving strategies  
- model how to use problem solving strategies  
- provide real world situations  
- value feedback given for invented strategies/representations used  
- teach students to focus on steps taken to solve the problem, not necessarily focusing on the answer | - create multiple representations to explain their thinking (pictures, words, number sentences, tables, & graphs)  
- learn to self-check - Does my answer make sense? (i.e. 5-3 ≠ 8, because my answer should be less than 5)  
- visualize problems |

### Non-examples

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| - do not model or allow time to practice  
- provide rigid teacher centered environment  
- limit variety of questions and provide no real-world connection  
- do not provide a safe environment for students to share invented strategies/representations  
- have a lot of emphasis on the answer  
- only teach by teacher led discussion | - lack the understanding that math relates to and is used in everyday life  
- use paper/pencil tasks only  
- are unaware of strategies and relationships  
- are confused with tasks and strategies  
- are unable to explain their thinking  
- give unreasonable answers |
## Grades K-1 Math Practices

**MP 3 Construct viable arguments and critique the reasoning of others**

<table>
<thead>
<tr>
<th>Definition</th>
<th>Student Friendly</th>
</tr>
</thead>
</table>
| Students can explain to others how they solved the problem. Students can listen and respond and critique the thinking and reasoning of others. | I can decide if my peers’ answers make sense.  
I can construct, explain and prove my answer. |

### Examples

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| ● create a safe environment for risk taking and critiquing with respect  
● use feedback to model desired student discourse  
● provide complex, rigorous tasks that foster deep thinking  
● plan effective questions, student grouping and time for student discourse  
● probe students to explain their thinking  
● model how to talk about and share solutions  
● model how to be a respectful participant (speaker or listener) when others are sharing  
● encourage the use of mathematical  | ● ask questions for clarification of another student’s thinking  
● analyze others arguments  
● use examples and counterexamples  
● observe mathematical patterns and make reasonable arguments  
● use objects, drawings, diagrams, and actions to support their thinking  
● use mathematical vocabulary |

### Non-examples

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| ● do not promote discourse or conversation among students through feedback  
● lead discourse and not students  
● do not foster risk taking and respect for the thinking of others in their classroom  
● ask only simple questions  
● attend only to the answer not the process  
● use language that is not precise (not mathematical)  | ● accept answers of others without questioning  
● respond inappropriately to the thinking of others  
● are unaware of mathematical patterns and unable to support their thinking  
● use only one method to support their thinking  
● use vague and imprecise mathematical language |
**Grades K-1 Math Practices**

**MP4 Model with Mathematics**

<table>
<thead>
<tr>
<th>Definition</th>
<th>Student Friendly</th>
</tr>
</thead>
</table>
| Students can find key information and use it to make a representation to draw conclusions and determine if it makes sense. | • I can use objects or pictures to show my thinking.  
• I can use equations to represent my thinking. |

“Does my model/representation match the problem?”  
“Does my answer make sense?”

<table>
<thead>
<tr>
<th>Examples</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Teachers</strong></td>
<td><strong>Students</strong></td>
</tr>
</tbody>
</table>
| • allow time for the process to take place  
• model the process not the product  
• make appropriate tools available  
• create a safe environment where risk taken is valued  
• provide meaningful, real-world, authentic, performance-based tasks  
• allow for discourse and investigation | • realize they use mathematics to solve real-world situations  
• use number relationships to draw conclusions (if 5+5=10 then 6+4=10)  
• determine if the solution makes sense and make corrections if needed  
• understand the tasks being asked |

<table>
<thead>
<tr>
<th>Non-examples</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Teachers</strong></td>
<td><strong>Students</strong></td>
</tr>
</tbody>
</table>
| • provide lack of modeling or time to practice  
• limit tools  
• use rigid teacher centered environment  
• use limited questions and provide no real-world connection  
• only teach through teacher led discussion | • lack understanding that math relates and is used in everyday life  
• use paper/pencil tasks only  
• are unaware of strategies and relationships  
• are confused with the tasks and strategies  
• are unable to explain their thinking |
## Grades K-1 Math Practices

### MP5 Use Appropriate Tools Strategically

<table>
<thead>
<tr>
<th>Definition</th>
<th>Student Friendly</th>
</tr>
</thead>
</table>
| Students can choose the appropriate tools to solve the given problem. They can use the appropriate tool for the situation at hand. | - I can choose the appropriate math tool to solve a problem.  
- I can use math tools to show how I solved a problem.  
- I can use math tools to help me solve a problem. |

### Examples

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| - model use of appropriate tools  
- provide opportunities for students to discover which tool is appropriate and why  
- compare/contrast the effectiveness of the tools  
- have a variety of tools available and encourage use of these tools (number lines, pictures, counting jars, tiles, number writing practices)  
- facilitate discussion with student experiences using tools (the why or why not of each tool) | - choose appropriate tools to solve a given problem  
- recognize the usefulness and limitations of different tools (number lines, pictures, counting jars, tiles, number writing practices) |

### Non-examples

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| - only provide modeling for one tool  
- provide limited time for discovery of tools  
- provide little or no discussion regarding efficiency of tools  
- do not provide instruction on how to use each tool (tape measurer)  
- do not refer to manipulatives as tools (instead of toys) | - use math tools as toys  
- do not pick the tool that is most appropriate  
- do not verbalize their rationale for use of the tool chosen |
### Grades K-1 Math Practices

#### MP6 Attend to Precision

**Definition**
Student can understand and use math vocabulary. Student can solve problems accurately and talk about their thinking. They can work carefully and check their work.

“Is my answer correct?”
“How can I prove it?”

**Student Friendly**
- I can work carefully and check my work.
- I can understand and use math vocabulary.
- I can solve problems accurately and talk about my thinking.

---

### Examples

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>• use mathematical language, and or charts, labeling shelves with pictures and words</td>
<td>• use math language</td>
</tr>
<tr>
<td>• link new vocabulary to common vocabulary</td>
<td>• show thinking with manipulatives, pictures, symbols, verbally</td>
</tr>
<tr>
<td>• model using manipulatives, pictures, symbols or verbally</td>
<td>• double check work</td>
</tr>
<tr>
<td>• explain/think aloud how to check work</td>
<td>• show thinking in more than one way</td>
</tr>
<tr>
<td>• ask open ended questions</td>
<td>• partner chat to think about their work</td>
</tr>
<tr>
<td>• encourage fruitful mistakes</td>
<td>• work toward efficient thinking</td>
</tr>
<tr>
<td>• are open to divergent thinking</td>
<td></td>
</tr>
<tr>
<td>• value the process, not just the product</td>
<td></td>
</tr>
</tbody>
</table>

---

### Non-examples

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>• tell how to solve the problem</td>
<td>• only try one strategy</td>
</tr>
<tr>
<td>• show only one way</td>
<td>• only try one tool</td>
</tr>
<tr>
<td>• do not question students reasoning</td>
<td>• show only product, not the process</td>
</tr>
<tr>
<td>• teach skills in isolation</td>
<td>• work in isolation</td>
</tr>
<tr>
<td>• do not provide opportunity for classroom discourse</td>
<td></td>
</tr>
</tbody>
</table>
Grades K-1 Math Practices

**MP 7 Look for and make use of structure**

<table>
<thead>
<tr>
<th>Definition</th>
<th>Student Friendly</th>
</tr>
</thead>
</table>
| Students use prior knowledge (what they already know) to solve new problems. They can break down complex problems into simpler, more manageable chunks. Students can recognize and understand the patterns they see in problem situations. | ● I can find patterns in numbers.  
● I can use patterns in numbers to solve problems.  
● I can take apart and put numbers back together.  
● I can break down problems into easier parts. |

**Examples**

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| ● use open ended questioning  
● are quiet and allow time for student discussion  
● foster persistence/stamina in problem solving by modeling and practice  
● provide students with tasks where they can look for structures and patterns | ● look for and identify patterns in numbers  
● use skills previously learned to solve new problems  
● break down complex problems into simpler tasks  
● understand how others solved the problem differently |

**Non-examples**

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| ● lead lessons without student discussion or questioning  
● provide one type of problem repeatedly  
● explain without modeling  
● provide clarification and explanations quickly | ● do not see the patterns in numbers  
● use a skill in isolation only  
● do not transfer a skill to a new situation  
● attempt to solve complex problem all at once  
● do not understand a different explanation from your own |
# Grades K-1 Math Practices

## MP8 Look for and express regularity in repeated reasoning

<table>
<thead>
<tr>
<th>Definition</th>
<th>Student Friendly</th>
</tr>
</thead>
</table>
| Students see mathematical patterns when solving problems. Students are able to make generalizations based on the patterns they see. | - I can solve problems by looking for patterns that repeat and use those patterns to solve other problems.  
- I can make generalizations based on patterns. |

### Examples

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| - provide rich and varied tasks that allow students to generalize relationships and methods, and build on prior mathematical knowledge  
- provide adequate time for exploration, dialogue and reflection, peer collaboration  
- ask deliberate questions that enable students to reflect on their own thinking periodically throughout the process  
- create strategic and intentional check-in points during student work time | - identify patterns and make generalizations  
- continually evaluate reasonableness of intermediate results  
- maintain oversight of the process  
- search for, identify, and use short-cuts |

### Non-examples

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| - provide one type of problem  
- provide limited problems for exploration  
- isolate individual skills  
- only assess at the end of unit  
- lead lessons with no discussion | - quickly answer without justification  
- are unable to make connections  
- view tasks in isolation  
- do not see any repeated patterns in their work |
Grades 2-3 Math Practices

**MP1 Makes sense of problems and perseveres in solving them**

<table>
<thead>
<tr>
<th>Definition</th>
<th>Student Friendly</th>
</tr>
</thead>
</table>
| Students can explain to themselves the meaning of a problem and look for entry points to a solution. They can plan a solution pathway— not jump into a solution attempt. Students can use prior knowledge to plan and solve, continually asking, “Does it make sense?” They will monitor and evaluate progress and change course if necessary. Students will check work using different methods/strategies and explain relationships between equations, graphs, diagrams, etc. | - I can explain/understand what the problem is asking.  
- I can make a plan.  
- I can make a plan using what I already know.  
- I can check to be sure that my answer makes sense.  
- I can change my plan if it isn’t working out.  
- I can solve my problem in different ways.  
- I can use representations to support my solution. |

**Examples**

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| - provide open-ended, rich problems  
- ask probing questions  
- provide opportunities for reflection (i.e. math journals)  
- provide rubrics for assessment and self-reflection  
- use manipulatives and technology to support strategies  
- display and discuss student work samples to show a variety of strategies  
- provide opportunities for student discourse and collaboration (i.e. Think-Pair-Share)  
- promote a safe environment to learn where students feel comfortable taking risks | - thoughtfully read information given - What do I notice?  
- highlight key words, numbers, and phrases within the question to help make sense of the problem  
- ask themselves if they need other information to solve  
- choose efficient strategies (plan) to solve  
- draw pictures, use representations, use manipulatives to create a plan  
- reflect to see the reasonableness of answers  
- collaborate by talking with peers about math  
- communicate their thinking orally and in writing |

**Non-examples**

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| - use limited math and language vocabulary  
- have students working only independently  
- teach a single algorithm as the “way” to solve  
- always have a quiet classroom  
- don’t provide time to share | - work with homework and classwork in a “drill and kill” fashion  
- do not use math manipulatives to demonstrate understanding  
- don’t share their thinking  
- give only one strategy to solve a problem  
- are required to solve problems a certain way |
### Grades 2-3 Math Practices

#### MP6 Attend to Precision

<table>
<thead>
<tr>
<th>Definition</th>
<th>Student Friendly</th>
</tr>
</thead>
</table>
| Precise clear communication of mathematical process and reasoning; includes work, algorithms, language, etc. | - I can work carefully and check my work.  
- I can understand and use the correct math vocabulary.  
- I can solve problems accurately and talk or write about my thinking  
- I can use appropriate labels with my answers. |

#### Examples

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| - understand precise definitions and context  
- use appropriate vocabulary throughout instruction  
- correct miscues  
- clarify vocabulary  
- display appropriate math language | - communicate with precision  
- calculate efficiently and accurately  
- explain mathematical reasoning using precise language, tools, pictures, labels, etc.  
- organize thinking  
- ask purposeful questions  
- check answers for accuracy  
- work towards clarity in discussions by carefully formulating an explanation |

<table>
<thead>
<tr>
<th>Non-examples</th>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| - do not understand the precise meaning of important mathematical vocabulary  
- rush thinking  
- limit resources  
- do not support and provide opportunities for student engagement | - have sloppy/incomplete labeling  
- show inaccurate computation  
- overuse the words “it” or “thing”  
- use lots of everyday language  
- lack explanation |
Grades 2-3 Math Practices

**MP2 Reason abstractly and quantitatively**

<table>
<thead>
<tr>
<th>Definition</th>
<th>Student Friendly</th>
</tr>
</thead>
</table>
| Students make sense of the quantities and use reasoning skills to understand relationship to the problem at hand. Students reason and recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. Students can solve problems in context. They can also pull out of a context to solve or represent with a number pattern. | • I can make sense of the problem.  
• I can write an equation that matches the problem.  
• I can use numbers, pictures, words, equations to show my thinking.  
• I can prove that my answer makes sense. |

### Examples

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>• encourage and model reasoning and making sense of problems</td>
<td>• develop habits of reasoning and creating of representations of the problem at hand</td>
</tr>
<tr>
<td>• use concrete models to foster habits of reasoning and create representations</td>
<td>• consider the units involved</td>
</tr>
<tr>
<td>• focus on the meaning of quantities - pairing visual images to representations and symbols</td>
<td>• attend to the meaning of quantities</td>
</tr>
<tr>
<td>• connect meaning of quantities to computations and properties of operations</td>
<td>• make sense of quantities and their relationship in problem solving</td>
</tr>
<tr>
<td></td>
<td>• understand the meaning of quantities and are flexible in the use of operations and their properties</td>
</tr>
<tr>
<td></td>
<td>• THINK - Engage - What makes sense?</td>
</tr>
</tbody>
</table>

### Non-examples

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>• use a lot of drill and practice worksheets</td>
<td>• rush through problems with little time spent in discussion or questioning</td>
</tr>
<tr>
<td>• focus on mostly on memorization rather than reasoning skills</td>
<td>• don’t make sense of quantities</td>
</tr>
<tr>
<td>• don’t allow talking</td>
<td>• don’t try to contextualize or decontextualize to solve problems</td>
</tr>
<tr>
<td>• promote the philosophy, “just do the work… quietly”</td>
<td>• do not represent math symbolically</td>
</tr>
<tr>
<td>• have an attitude of “my way or the highway”</td>
<td></td>
</tr>
</tbody>
</table>
Grades 2-3 Math Practices

**MP3 Construct viable arguments and critique the reasoning of others**

<table>
<thead>
<tr>
<th>Definition</th>
<th>Student Friendly</th>
</tr>
</thead>
</table>
| Students need to articulate their reasoning and identify what works and what doesn’t in the reasoning of others. Their conversations should show understanding of the concept to build a logical progression of statements, as well as recognizing and using counterexamples. Students should justify their conclusions, communicate them to others and respond to the arguments of others. | - I can explain my thinking and support it with math words, symbols and visuals.  
- I can listen to the reasoning of others and decide if it is reasonable.  
- I can apply the reasoning of others to a math problem. |

**Examples**

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| - create a safe environment for risk-taking and critique with respect  
- model desired student discourse  
- provide complex, rigorous tasks that foster understanding/opportunities for discussion  
- plan and use effective questions  
- use effective grouping of students  
- provide time for student discourse | - ask questions to clarify misconceptions  
- use examples and nonexamples  
- make sense in their reasoning and explain their thinking clearly  
- compare two plausible arguments, distinguish correct reasoning from flawed reasoning, and explain why an argument is flawed |

**Non-examples**

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| - is seen as the sage on the stage  
- dictates one way to find a solution | - learn and work silently  
- focus on one person’s work and one way to solve a problem  
- hurt others feelings due to personal attack  
- argue inappropriately about work samples |
# Grades 2-3 Math Practices

## MP4 Model with Mathematics

<table>
<thead>
<tr>
<th><strong>Definition</strong></th>
<th><strong>Student Friendly</strong></th>
</tr>
</thead>
</table>
| Students can apply the mathematics they know to solve real world problems. This includes writing an equation to describe and solve a situation. | - I can use geometric figures, pictures, or physical objects or diagrams such as a number line, a table or graph to represent a real world problem.  
- I can show my work in many ways.  
- I can use expressions or equations to represent my thinking. |

## Examples

### Teachers
- allow time for the process to take place (model, make graphs…)
- model desired behaviors (think alouds) and thought processes (questioning, revision, reflection/written)
- make appropriate tools available for students to select from
- create an emotionally safe environment where risk taking is valued
- provide meaningful, real world, authentic, performance-based tasks (non-traditional word problems)
- encourage and model discourse
- encouraging students to investigate their own mathematical curiosities

### Students
- realize they use mathematics (numbers and symbols) to solve/work out real-life situations
- analyze relationships to draw conclusions
- interpret mathematical results in context
- show evidence that they can use their mathematical results to think about a problem and determine if the results are reasonable; if not, go back and look for more information
- make sense of the mathematics
- simplify a complicated situation
- identify important quantities

## Non-examples

### Teachers
- limit time for students to process
- provide a lack of opportunity for discourse
- limit or allow no tools to be available for students to demonstrate understanding
- do not model
- demonstrate lack of planning
- provide students with low level traditional word problems

### Students
- work completely independent of others often
- do not provide an explanation with their answer
- do not use models or equations to represent their thinking
- write simple answers
- do not see mathematics as a tool to solve real world problems
Grades 2-3 Math Practices

**MP5 Use Appropriate Tools Strategically**

<table>
<thead>
<tr>
<th>Definition</th>
<th>Student Friendly</th>
</tr>
</thead>
</table>
| Students consider the available tools when solving a mathematical problem. Tools should be a variety of manipulatives and technology. Students are sufficiently familiar with tools appropriate for the task. | - I can choose and use the most appropriate tool for solving a problem.  
- I am familiar with lots of different tools I can use to solve math problems. |

**Examples**

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| - maintain knowledge of appropriate tools  
- effectively model use of the tools available, their benefits and limitations  
- model a situation where the decision needs to be made as to which tool should be used  
- compare/contrast effectiveness of tools  
- make available and encourage use of a variety of tools | - choose the appropriate tool to solve a given problem and deepen their conceptual understanding (paper/pencil, ruler, base 10 blocks, compass, protractor)  
- choose the appropriate technological tool to solve a given problem and deepen their conceptual understanding (e.g., spreadsheet, geometry, software, calculator, web 2.0 tools)  
- compare the efficiency of different tools  
- recognize the usefulness and limitations of different tools |

**Non-examples**

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| - do not show awareness of effective tools  
- limit the availability of tools to what they would choose | - relies solely on paper and pencil as a tool  
- relies solely on mental math when the task warrants more |
**Grades 2-3 Math Practices**

**MP7 Look for and make use of structure**

<table>
<thead>
<tr>
<th>Definition</th>
<th>Student Friendly</th>
</tr>
</thead>
</table>
| Students look closely to discern a pattern or structure. Examples would be commutative property, associative property, decomposing a number with place value within addition and subtraction strategies, mentally adding/subtracting 10 and 100. | • I can identify patterns within a problem and use efficient strategies to solve.  
• I can see and understand how numbers and shapes are organized and put together as parts and wholes.  
• I can break down complex problems into smaller chunks to solve accurately. |

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Teachers</strong></td>
</tr>
</tbody>
</table>
| • facilitate learning by using open-ended questioning to assist students in exploration  
• carefully select tasks to promote opportunities to look for and identify patterns  
• guide students to use patterns to make generalizations  
• provide time for student discussion and processing (i.e. wait time)  
• foster persistence and stamina in problem solving through practice and modeling |
| **Students** |
| • look closely to discern patterns and structure  
• recognize, reflect on and interpret patterns and structures using number grid, number line, number bonds, place value chart and ten-frames  
• use previously learned skills and strategies to solve new problems and tasks  
• decompose numbers into more workable numbers such as hundreds, tens and ones  
• decompose shapes into more workable shapes |

<table>
<thead>
<tr>
<th>Non-examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Teachers</strong></td>
</tr>
</tbody>
</table>
| • do not provide mathematical structures to help students understand the mathematics  
• use a lot of low level problem types  
• are not thoughtful about the types of relevant practices provided to students to identify patterns  
• teach through teacher led discussion most of the time, tell students what the relationships are instead of discovery and inquiry  
• limit student participation |
| **Students** |
| • are not engaged or involved  
• do not make connections or use previously learned strategies  
• do not use mathematical patterns or structures to make sense of the problem  
• cannot apply mathematical methods/algorithm to not traditional problems |
Grades 2-3 Math Practices

**MP8 Look for and express regularity in repeated reasoning**

<table>
<thead>
<tr>
<th><strong>Definition</strong></th>
<th><strong>Student Friendly</strong></th>
</tr>
</thead>
</table>
| Students notice if calculations are repeated, and look both for general methods and patterns to use as shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process while attending to details. They continually evaluate the reasonableness of the intermediate results. | • I can solve problems by looking for patterns that repeat and rules that can apply to other problems.  
• I look carefully at the entire problem and pay attention to details.  
• I repeatedly ask myself, “Does my answer make sense?”  
• I am looking for patterns to help me solve my work more efficiently. |

<table>
<thead>
<tr>
<th><strong>Examples</strong></th>
<th><strong>Teachers</strong></th>
<th><strong>Students</strong></th>
</tr>
</thead>
</table>
| • provide rich and varied tasks that allow students to generalize relationships and methods that will build on prior mathematical knowledge | • continually evaluate the reasonableness of results  
• search for generalizations to identify and use patterns as short-cuts  
• notice repetition and regularity of patterns, for example odd and even patterns or patterns of operations  
• attend to details |
| • provide adequate time for exploration, dialogue and reflection, peer collaboration  
• ask deliberate questions that enable students to reflect on their own thinking periodically throughout the process  
• create strategic and intentional check in points during student work time | |

<table>
<thead>
<tr>
<th><strong>Non-examples</strong></th>
<th><strong>Teachers</strong></th>
<th><strong>Students</strong></th>
</tr>
</thead>
</table>
| • provide only repetitive skill and drill practice that do not promote recognition of patterns for students  
• provide only knowledge-based “recall” questioning  
• do not expect students to justify their thinking with reasoning or generalizations  
• do not provide opportunities for students to make connections between addition and subtraction as different ways to solve the same problem  
• do not provide opportunities for students to make connections between multiplication and division as different ways to solve the same problem | • give limited response  
• do not apply generalizations or patterns in their responses  
• do not make connections to patterns and relationships  
• do not see the relationship between addition and subtraction or multiplication and division |
### Grades 4-6 Math Practices

**MP1 Makes sense of problems and perseveres in solving them**

<table>
<thead>
<tr>
<th>Definition</th>
<th>Student Friendly</th>
</tr>
</thead>
</table>
| Students will interpret and analyze to find the meaning of the problem/make sense of the problem. They will create a plan to solve the problem, monitor their progress and change the approach if necessary. | - I can explain/understand what the problem is asking.  
- I can make a plan.  
- I can get unstuck when I am stuck.  
- I can change my plan if it isn’t working out.  
- I can monitor my work.  
- I can prove that my answer makes sense.  
- I can show that my representations in my work support my solution. |

### Examples

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| - provide open ended and rich problems  
- ask probing questions  
- model multiple problem-solving strategies through *think aloud*  
- promote and value disclosure and collaboration  
- examine student responses (correct or incorrect) for understanding and multiple representations  
- promote a safe environment to learn in  
- view mistakes as learning opportunities  
- differentiate based on student needs  
- encourage self-monitoring | - thoughtfully read information given - What do I notice?  
- underline the question being asked  
- make sense of the problem by double underlining key direction words, circling information/numbers needed to solve  
- highlight key math vocabulary or terms in the problem  
- reread thoughtfully  
- ask themselves if they need other information to solve the problem  
- choose efficient strategies (Plan) to solve  
- draw math pictures, use representations, manipulatives to create a plan  
- check to see - is my answer reasonable? does it make sense of the question asked?  
- change plan if needed |

### Non-examples

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| - focus on telling the students “how to solve” rather than building problem solving strategies  
- simplify language so that students don’t have to think to solve the problem  
- use limited mathematical language | - work alone  
- have not developed a clear path for solving problem  
- are more interested in the “how to” than in understanding the bigger concepts and understanding why  
- do not show representations of their understandings |
Grades 4-6 Math Practices

**MP2 Reason abstractly and quantitatively**

<table>
<thead>
<tr>
<th>Definitions</th>
<th>Student Friendly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students understand what the numbers represent. They show how to represent</td>
<td>• I can make sense of the problem.</td>
</tr>
<tr>
<td>the problems using symbols and numbers. Students can develop more than one</td>
<td>• I can show my process to get a solution.</td>
</tr>
<tr>
<td>strategy or solution to a problem. Learners understand the relationships</td>
<td>• I can check my answer to see if it makes sense.</td>
</tr>
<tr>
<td>between problem scenarios and mathematical representation.</td>
<td>• I can use numbers, words and reasoning to help me make sense of problems.</td>
</tr>
<tr>
<td></td>
<td>• I understand what my solution means.</td>
</tr>
</tbody>
</table>

**Examples**

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>• develop opportunities for and model problem solving strategies</td>
<td>• estimate first to make more sense of the answer</td>
</tr>
<tr>
<td>• provide real world situations</td>
<td>• make sure the answer is reasonable</td>
</tr>
<tr>
<td>• makes connections between content areas</td>
<td>• create multiple ways to represent their problem solving (tables, pictures, words, symbols)</td>
</tr>
<tr>
<td>• value invented strategies and representations</td>
<td>• represent the unit</td>
</tr>
<tr>
<td>• give less emphasis to the answer</td>
<td></td>
</tr>
<tr>
<td>• help their learners understand the relationships between problem</td>
<td></td>
</tr>
<tr>
<td>scenarios and mathematical representation</td>
<td></td>
</tr>
</tbody>
</table>

**Non-examples**

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>• expect no explanation with the answer</td>
<td>• provide answers that are not labeled or do not make sense</td>
</tr>
<tr>
<td>• do not connect math to the real world or other content areas</td>
<td>• do not pull out numbers/information to effectively to solve the problem</td>
</tr>
</tbody>
</table>
### Grades 4-6 Math Practices

**MP3 Construct viable arguments and critique the reasoning of others**

#### Definitions
Students engage in active mathematical discourse, this might involve having students explain and discuss their thinking processes aloud or signaling agreement/disagreement with a hand signal. A teacher might post multiple approaches to a problem and ask students to identify plausible rationales for each approach.

#### Student Friendly
- I can make conjectures and critique of the mathematical thinking of others.
- I can construct, justify and communicate arguments by…
  1. considering context
  2. using examples and non-examples
  3. using objects, drawings, diagrams and actions
- I can critique the reasoning of others by…
  1. listening
  2. comparing arguments
  3. identifying flawed logic
  4. asking questions to clarify or improve arguments

#### Examples

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>create a safe environment for risk-taking and critiquing with respect</td>
<td>ask questions</td>
</tr>
<tr>
<td>use feedback to model desired student discourse</td>
<td>reason inductively and make plausible arguments</td>
</tr>
<tr>
<td>provide complex, rigorous tasks that foster deep thinking</td>
<td>use examples and counterexamples to prove their understanding or arguments</td>
</tr>
<tr>
<td>plan effective questions, student grouping and time for student discourse</td>
<td></td>
</tr>
<tr>
<td>probe students’ thinking to gain insight into their understanding</td>
<td>analyze others arguments</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Non-examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Teachers</strong></td>
</tr>
<tr>
<td>focus on just the answer</td>
</tr>
<tr>
<td>teach procedural methods without understanding</td>
</tr>
<tr>
<td>use a lot of “drill and kill” problem practice</td>
</tr>
<tr>
<td>provide feedback that does not provide any room for further discourse (i.e. right or wrong)</td>
</tr>
</tbody>
</table>
Grades 4-6 Math Practices

**MP4 Model with mathematics**

<table>
<thead>
<tr>
<th><strong>Definition</strong></th>
<th><strong>Student Friendly</strong></th>
</tr>
</thead>
</table>
| Students construct visual evidence using symbolic and graphical representations. They use pictures, numbers and words appropriate to the real world context of the problem. | - I can use geometric figures, pictures, or physical objects or diagrams such as a number line, tape diagram, table or graph to represent the problem.  
- I can show my work in multiple ways.  
- I can use expressions or equations to represent my thinking. |

**Examples**

<table>
<thead>
<tr>
<th><strong>Teachers</strong></th>
<th><strong>Students</strong></th>
</tr>
</thead>
</table>
| - provide open ended questions/problems that are meaningful, real world, authentic, performance-based tasks  
- demonstrate the process of modeling  
- share students’ examples  
- allow time for the process to take place (model, make graphs…)  
- provide many opportunities to connect and explain the different relationship between the different representations | - represent work in pictures (models), numbers and words  
- need many opportunities to connect and explain the different relationship between the different representations  
- show evidence that they can use their mathematical results to think about a problem and determine if the results are reasonable; if not, go back and look for more information |

**Non-examples**

<table>
<thead>
<tr>
<th><strong>Teachers</strong></th>
<th><strong>Students</strong></th>
</tr>
</thead>
</table>
| - provide lower level multiple choice questions with no room to solve problems  
- focus instruction on procedural method | - perform using algorithms only  
- use limited representation to explain reasoning  
- can’t explain, justify or model |
**Grades 4-6 Math Practices**

**MP5 Use appropriate tools strategically**

<table>
<thead>
<tr>
<th>Definition</th>
<th>Student Friendly</th>
</tr>
</thead>
</table>
| Students identify and use tools to scaffold their learning and increase understanding of concepts, problem-solving strategies, and to further exploration of possible solutions. They make good decisions about the appropriateness of the specific tool or tools to be used. | - I can choose the most appropriate tool for each given problem.  
- I can use math tools to show how I solved a problem.  
- I can use math tools to help me solve a problem.  
- I know HOW to use math tools. |

**Examples**

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| - model use of tools - ex. when is an appropriate time to use a calculator; when is a ruler needed? use square tiles to model arrays  
- build understanding of concepts through the use of models and manipulatives - moving from concrete to representation to abstract  
- model use of resources - how to use the mathematical dictionaries, online tools - to help develop math vocabulary and broaden understanding of math concepts  
- identify and model use of online tools | - use manipulatives to build representations and problem-solving strategies  
- identify math vocabulary - use math resources to determine meaning of unknown math terms  
- will choose the appropriate tool to help them problem solve in an efficient/strategic manner  
- strategically and thoughtfully use tools when appropriate in problem solving situation  
- demonstrate when and how to use math tools |

**Non-examples**

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| - use the same tool over and over and over again without utilizing a variety of different tools and/or teaching methods  
- allowing the use of prohibited tools (unless accommodations are needed)  
- do not allow students to make sense of needed tools based on the mathematics  
- do not have tools available for easy access for students to use when they need to use them | - repeated use of tool without understanding  
- use the tool when other methods are more efficient and appropriate (99-97 does not need a calculator)  
- do not make sense of the tool and if it is the best tool for the mathematics at hand |
Grades 4-6 Math Practices

**MP6 Attend to precision**

<table>
<thead>
<tr>
<th>Definition</th>
<th>Student Friendly</th>
</tr>
</thead>
</table>
| Students calculate accurately and efficiently. They use clear and concise communication, written & oral to explain their understanding and thinking. Students use correct mathematical vocabulary and symbols to communicate their thinking. | • I can work carefully and check my work.  
• I can understand and use math vocabulary.  
• I can solve problems accurately and talk and write about my strategies and solutions. |

<table>
<thead>
<tr>
<th>Examples</th>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| • create a classroom environment that is safe for risk taking, communication and evaluation of each other’s thinking  
• ask probing questions that require students to analyze their thinking and critically evaluate their reasoning  
• model mathematical language, model use of resources  
• value the process, not just the product  
• model/encourage think aloud & double checking of work | • represent their work to match what the problem is asking  
• talk to one another about their mathematical thinking, share ideas with one another, ask questions of one another  
• demonstrate mathematical fluency by efficiently, accurately and fluently adding, subtracting, multiplying and dividing  
• use math language/vocabulary appropriately  
• use resources to support understanding of math vocabulary |

<table>
<thead>
<tr>
<th>Non-examples</th>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
</table>
| • do not provide problems that allow for extended responses  
• focus on only one path to get an answer  
• do not understand the precise meaning of important mathematical vocabulary  
• rush thinking  
• limit resources  
• do not provide opportunity for student engagement | • work in isolation  
• are not discussing strategies and solutions with peers  
• are sloppy with their representations and work is incomplete missing labeling that shows understanding  
• have inaccurate computation  
• overuse of the words “it” and “thing”  
• lots of everyday language  
• lack explanation in their work |
### Grades 4-6 Math Practices

**MP7 Look for and make use of structure**

<table>
<thead>
<tr>
<th>Definition</th>
<th>Student Friendly</th>
</tr>
</thead>
</table>
| Students look for, interpret and identify patterns and structures. They see complicated things as single objects or being composed of several objects. Students make connections to skills and strategies previously learned to solve new problems/tasks independently and with peers. | *I can notice when calculations are repeated. Then, I can find more efficient methods.*
|                                                                           | *I can see and understand how numbers and shapes are organized and put together as parts and wholes.*
|                                                                           | *I can take complex problems into simpler, more manageable chunks.*
|                                                                           | *I can use the structure of mathematics to make sense of my thinking.*            |

**Examples**

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>● understand the properties of operations so they can help students make connections</td>
<td>● show how patterns emerge in rich problems</td>
</tr>
<tr>
<td>● show connections between types of numbers; fractions, decimals</td>
<td>● use repeated patterns to find a more efficient way of solving</td>
</tr>
<tr>
<td>● provide rich problems to look for repeated reasoning and connections</td>
<td>● use repeated patterns to show understanding of mathematics</td>
</tr>
</tbody>
</table>

**Non-examples**

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>● do not provide mathematical structures to help students understand the mathematics</td>
<td>● do not use mathematical patterns or structures to make sense of the problem</td>
</tr>
<tr>
<td>● use a lot of low level problem types</td>
<td>● are not able to make connections or use previous strategies.</td>
</tr>
<tr>
<td>● are not thoughtful about the types of relevant practice provided to students to identify patterns</td>
<td>● cannot apply mathematical methods/algorithms to non-traditional problems</td>
</tr>
<tr>
<td>● teach through teacher led discussion most of the time, tell students what the relationships are instead of discovery and inquiry</td>
<td>● are sitting for long time periods</td>
</tr>
<tr>
<td>● limit student participation</td>
<td>● are not engaged or show little involvement</td>
</tr>
</tbody>
</table>
### Grades 4-6 Math Practices

**MP8 Look for and express regularity in repeated reasoning**

<table>
<thead>
<tr>
<th>Definitions</th>
<th>Student Friendly</th>
</tr>
</thead>
</table>
| Students notice if calculations are repeated and look for general methods and shortcuts. They maintain oversight of the process while attending to details. Students continually evaluate the reasonableness of immediate results. | - I can solve problems by looking for patterns that repeat and rules that can apply to other problems.  
- I look carefully at the entire problem and pay attention to details.  
- I repeatedly ask myself, “Does my answer make sense?”  
- I am looking for patterns to help me solve my work more efficiently. |

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Teachers</strong></td>
</tr>
</tbody>
</table>
| - provide rich and varied tasks that allow students to generalize relationships and methods, and build prior mathematical knowledge  
- provide adequate time for exploration, dialogue and reflection, peer collaboration  
- ask deliberate questions that enable students to reflect on their own thinking periodically throughout the process  
- create strategic and intentional check in points during student work time | - ask: is there a pattern here?  
- wonder: how can I generalize this pattern?  
- search for and recognize there is a predictable pattern that will help me in my mathematical understanding  
- ask: is there a shortcut based on a repeated pattern, will it provide for more efficiency and accuracy? |

<table>
<thead>
<tr>
<th>Non-examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Teachers</strong></td>
</tr>
</tbody>
</table>
| - do not probe students’ to make observations about patterns  
- provide repetitive skill and drill practice  
- only ask knowledge-based “recall” questioning | - give limited response  
- do not apply generalizations or patterns in their responses  
- do not make connections to patterns and relationships  
- do not see the relationships in number or patterns |
**FHSD Standards for Mathematical Practice Look-for Tool**

The look-for tool was developed by the FHSD Course of Study Committee based off of the look-for tool created by the National Council of Supervisors of Mathematics (NCSM.)

“This look-for tool is a classroom resource that can be used as a non-evaluative assessment tool. The “look-fors” in this peer-observation tool are designed to assess the extent to which students are engaged in particular elements of the mathematical practices of the Common Core State Standards for mathematics. The tool provides suggested student responses for each of the mathematical practices as observational look-fors. These should be helpful in providing suggestions related to both planning and assessment for teachers, mathematical leaders and school based administrators.

The look-fors in this peer observation tool are designed to measure how well the instructor has integrated the essence of the principles of the Common Core for the mathematics into their classroom. There are eight key mathematical practices with specific look-fors for both student and teacher behaviors. Noting the occurrence and totality of these look-fors can be helpful in providing feedback and guidance for the instructor.” NCSM
**NOTE:** All indicators are not necessary for providing full evidence of practice(s). Each practice may not be evident during every lesson.

<table>
<thead>
<tr>
<th>Mathematics Practices</th>
<th>Student dispositions:</th>
<th>Teacher actions to engage students in Practices:</th>
</tr>
</thead>
</table>
| **1. Make sense of problems and persevere in solving them** | [ ] Understand the problem  
[ ] Use strategies/strategic thinking to plan a solution pathway  
[ ] Use patience and persistence to solve problem  
[ ] Use self-evaluation and redirections  
[ ] Communicate both verbally and written  
[ ] Be able to deduce what is a reasonable solution  
[ ] Perseverance through multiple-step problems | Comments: |
| **2. Reason abstractly and quantitatively** | [ ] Communicate with precision-oral and written  
[ ] Use mathematics concepts, symbols and vocabulary appropriately  
[ ] Attend to units labeling/tools accurately  
[ ] Carefully formulate explanations  
[ ] Ensure reasonableness of answers  
[ ] Calculate accurately and efficiently | Comments: |
| **3. Construct viable arguments and critique the reasoning of others** | [ ] Create and connect multiple representations (tables, pictures, words, symbols, graphs)  
[ ] Estimate first/answer reasonable  
[ ] De-contextualize  
[ ] Using context to think about a problem  
[ ] Attend to the meaning of the quantities, not just computing | Comments: |
| **6. Attend to precision** | [ ] Teacher models and encourages students to think aloud/talk aloud  
[ ] Guided Inquiry including teacher gives problem, students work together to solve problems, and debriefing time for sharing and comparing strategies  
[ ] Promote mathematical terminology/vocabulary  
[ ] Give room to discuss why wrong answers are wrong | Comments: |
<table>
<thead>
<tr>
<th>Mathematics Practices</th>
<th>Student dispositions:</th>
<th>Teacher actions to engage students in Practices:</th>
</tr>
</thead>
</table>
| 4. Model with Mathematics | - Apply mathematics (numbers and symbols) to solve/work out real-life situations  
- Show evidence that they can use their mathematical results to think about a problem and determine if the results are reasonable  
- Identify important quantities in a practical situation  
- Be comfortable making assumptions and approximations to simplify a complicated situation  
- Analyze relationships to draw conclusions | - Allow time for the process to take place (model, make graphs, etc.)  
- Make appropriate tools available  
- Provide meaningful, real world, authentic and performance-based tasks (non-traditional work problems)  
- Provide opportunities to investigate and integrate cross-curricularly  

**Comments:** |
| 5. Use appropriate tools strategically | - Recognize the usefulness and limitations of different tools  
- Choose the appropriate tool to solve a given problem and deepen their conceptual understanding (e.g., paper/pencil and manipulatives)  
- Choose the appropriate technological tool to solve a given problem and deepen their conceptual understanding (e.g., spreadsheet, geometry software, calculator, web 2.0 tools)  

**Comments:** | - Maintain knowledge of appropriate tools  
- Effective modeling of the tools available, their benefits and limitations  
- Make available and encourage use of a variety of tools  

**Comments:** |
| 7. Look for and make use of structure | - Recognize, reflect on, and interpret patterns and structures  
- Make connections to skills and strategies previously learned to solve new problems/tasks  
- Use the structure of mathematics to solve simpler forms of the original problem  
- Breakdown complex problems into simpler, more manageable chunks  
- Be able to “step back” / shift perspective  

**Comments:** | - Facilitate learning by using open-ended questioning to assist students in exploration  
- Careful selection of tasks that allow for students to discern structures or patterns to make connections  
- Be quiet and allow time for student discussion and processing in place of fixed rules or definitions  

**Comments:** |
| 8. Look for and express regularity in repeated reasoning | - Maintain oversight of the process by continually evaluating the reasonableness of intermediate results  
- Search for generalizations to identify and use short-cuts  

**Comments:** | - Provide rich and varied tasks that allow students to generalize relationships and methods, and build on prior mathematical knowledge  
- Provide adequate time for exploration, dialogue and reflection, peer collaboration  
- Ask deliberate questions that enable students to reflect on their own thinking periodically throughout the process  
- Create strategic and intentional check in points during student work time  

**Comments:** |
Mathematical Practice Classroom Visuals

The FHSD Course of Study Committee designed and approved the visuals. These visuals will be used in the Forest Hills classrooms as reminders to administrators, teachers, and students as to the importance of the practices. The practices tell us “how” the mathematics should look in the classroom.

Ideas of how to use the visuals
- display the posters in the classroom, reference them with students
- display student work next to the posters in the classroom
- use the visuals on assignments as a reminder for students of the expectations in the classroom
- post pictures of students “caught” in the act of one of the practices
- use the visuals on parent newsletters to acquaint parents with the practices

<table>
<thead>
<tr>
<th>Understand and Persevere</th>
<th>Reason Abstractly</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP1: I can make sense of problems and not give up when trying to solve them.</td>
<td>MP2: I can use numbers, words, and visuals to help me make sense of problems.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Construct and Critique</th>
<th>Model with Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP3: I can explain my thinking, listen to and critique the reasoning of others.</td>
<td>MP4: I can use math I know to solve problems in real life.</td>
</tr>
</tbody>
</table>
Strategically Use Tools

MP5: I can choose the appropriate tools to help me solve problems.

Attend to Precision

MP6: I can solve problems accurately and clearly talk about my thinking.

Make Use of Structure

MP7: I can use what I already know to break down challenging problems into easier parts.

Use Repeated Reasoning

MP8: I can find and use mathematical patterns to help me solve a problem.
Section 3: FHSD Mathematical Teaching Habits (NCTM)

The teaching of mathematics is complex and requires teachers to have a deep understanding of the mathematical knowledge they are expected to teach. (Deborah Ball, NCTM) Teachers are expected to have a clear view of how student learning of that mathematics develops and progresses across grades. Also required of teachers is the need to be skilled at teaching in ways that effectively develop mathematics learning for all students. (Daro, Mosher, & Corcoran 2011)

Research conducted from cognitive science (National Research Council 2012) and mathematics education (Donovan and Bransford 2005) supports characterization of mathematics as an active process. Students build on his or her own mathematical knowledge from personal experiences, coupled with feedback from peers, teachers, and other adults, and themselves. The research has identified a number of principles of learning that provide the foundation for effective mathematics teaching.

The Eight Mathematics Teaching Practices (renamed, “Habits” by the FHSD course of study) provide a framework for strengthening the teaching and learning of mathematics. These habits represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics. The eight practices are based on the research mentioned above. Forest Hills Course of Study Committee has agreed that these practices should guide our professional learning opportunities for our teachers. (NCTM)

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

Based on the book, Principles to Actions: Ensuring Mathematical Success for All, from NCTM. For a copy of this book please contact the FHSD curriculum department.
1. Establish Mathematics Goals to Focus Learning

Effective teaching of mathematics

- establishes clear goals for the mathematics that students are learning
- situates goals within learning progressions
- uses the goals to guide instructional decisions

<table>
<thead>
<tr>
<th>Teacher Actions</th>
<th>Student Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Establish clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit.</td>
<td>Engage in discussions of the mathematical purpose and goals related to their current work in the mathematical classroom (What are we learning? Why are we learning it?)</td>
</tr>
<tr>
<td>Identify how goals fit within a mathematics learning progression.</td>
<td>Use the learning goals to stay focused on their progress in improving their understanding of mathematics content and proficiency in using mathematical practices.</td>
</tr>
<tr>
<td>Discuss and refer to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning.</td>
<td>Connect their current work with the mathematics that they studied previously and see where the mathematics is going.</td>
</tr>
<tr>
<td>Use the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction.</td>
<td>Assess and monitor toward the mathematics learning goals.</td>
</tr>
</tbody>
</table>

Instructional Research Based Strategies

- “formulating clear, explicit learning goals sets the stage for everything else” (Heibert)
- “goals should be situated within mathematical learning progressions and connected to the big ideas” (Daro, Mosher and Corcoran)
- “classrooms where students understand the learning expectations for their work perform at higher levels than those classrooms where it is unclear” (Marzano, Haystead, Hattie)
- “student friendly goals can be discussed within a lesson so that students see value in and understand the purpose of their work” (Black and William, Marzano)
- “as teachers establish specific goals and consider the connections to broader math topics, they become more prepared to use goals to make decisions during instruction” (Heibert)

Important Instructional Considerations

- goals communicate what students will understand based on instruction
- goals identify mathematical practices that students are learning to use more proficiently
- goals should not just be reiteration of standards but should be linked to curriculum and student learning
- goals should be situated within the mathematical landscape to support opportunities to build connections so that students see how ideas build
- mathematical purpose of the lesson should not be a mystery to students
Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.

Instructional Materials Statement
FHSD has adopted Eureka Math as the Tier 1 curriculum for grades K-6. Use of this curriculum, Investigations and other standards based resources should be used as advised in the FHSD curriculum maps. The learning target statements in the curriculum maps identify the learning targets for each standard.

Student Materials

Instructional Resources
FHSD Curriculum Maps
Classroom Assessment for Student Learning, Doing It Right-Using It Well, Jan Chappuis and Rick Stiggins
2. Implement tasks that promote reasoning and problem solving

Effective teaching of mathematics

- engages students in solving and discussing tasks that promote mathematical reasoning and problem solving
- allows for multiple entry points and varied solution strategies

<table>
<thead>
<tr>
<th>Teacher Actions</th>
<th>Student Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motivate students’ learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding.</td>
<td>Persevere in exploring and reasoning through tasks.</td>
</tr>
<tr>
<td>Select tasks that provide multiple entry points through the use of varied tools and representations.</td>
<td>Take responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas.</td>
</tr>
<tr>
<td>Pose tasks on a regular basis that require a high level of cognitive demand. Support students in exploring tasks without taking over student thinking.</td>
<td>Use tools and representations as needed to support their thinking and problem solving.</td>
</tr>
<tr>
<td>Encourage students to use varied approaches and strategies to make sense of and solve tasks.</td>
<td>Accept and expect that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another.</td>
</tr>
</tbody>
</table>

Instructional Research Based Strategies

- “...mathematical tasks are viewed as placing higher-level cognitive demands on students when they allow students to engage in active inquiry and exploration or encourage students to use procedures in ways that are meaningfully connected with concepts or understanding.” (Smith and Stein, 1998)
- “Tasks that encourage students to use procedures, formulas, or algorithms in ways that are not actively linked to meaning, or that consist primarily of memorization or the reproduction of previously memorized facts, are viewed as placing lower-level cognitive demands on students.” (Smith and Stein, 1998)
- a task should provide students with the opportunity to engage actively in reasoning, sense making, and problem solving so that they develop a deep understanding of mathematics (NCTM)

Important Instructional Considerations

- Provide rich, open ended tasks that allow for multiple solutions
- Model and use visual supports
- Use [Webb’s Depths of Knowledge](http://www.webb.org/knowledge/) to guide high-level of task selection

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Construct viable arguments and critique the reasoning of others.
3. Use appropriate tools strategically.
4. Attend to precision.
Instructional Materials Statement
FHSD has adopted Eureka math as the Tier 1 curriculum for grades K-6. Use of this curriculum, Investigations and other standards based resources should be used as advised in the FHSD curriculum maps.

Student Materials
Eureka Math-student edition

Instructional Resources
Eureka Math
FHSD Curriculum Maps
Document camera for sharing multiple strategies & student examples
3. Use and Connect Mathematical Representations

Effective teaching of mathematics
- engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tool for problem solving

<table>
<thead>
<tr>
<th>Teacher Actions</th>
<th>Actions of Student</th>
</tr>
</thead>
<tbody>
<tr>
<td>Select tasks that allow students to decide which representations to use in making sense of the problems.</td>
<td>Use and explore multiple forms of representation to make sense of and understand mathematics.</td>
</tr>
<tr>
<td>Allocate substantial instructional time for students to use, discuss, and make connections among representations.</td>
<td>Describe and justify mathematical understanding and reasoning using effective representations / drawings, diagrams, words, ....</td>
</tr>
<tr>
<td>Introduce forms of representations that can be useful to students.</td>
<td>Make choices about which forms or representations to use as tools for solving problems.</td>
</tr>
<tr>
<td>Ask students to make math drawings or use other visual supports to explain and justify their reasoning.</td>
<td>Make sense of problems through use of tables, drawings, diagrams and other representations.</td>
</tr>
<tr>
<td>Focus students’ attention on the structure or essential features of mathematical ideas that appear, regardless of the representation.</td>
<td>Connect mathematical ideas and concepts to real-world situations and contexts.</td>
</tr>
<tr>
<td>Design ways to elicit and assess students’ abilities to use representations meaningfully to solve problems.</td>
<td>Consider advantages or suitability of using alternate representations when problem solving.</td>
</tr>
</tbody>
</table>

Instructional Research Based Strategies
- When students learn to represent, discuss, and make connections among mathematical ideas in multiple forms, they demonstrate a deeper understanding and enhanced problem-solving abilities. (Fuson, Kalchman, & Bransford 2005; Lesh, Post and Behr 1987)
- The depth of understanding is related to the strength of connections among mathematical representations that students have internalized. (Pape and Tchoshanov 2001; Webb, Boswinkel and Dekker 2008)
- Visual representations are of particular importance in the mathematics classroom, helping students to advance their understanding of mathematical concepts and procedures, make sense of problems and engage in mathematical discourse. (Arcavi 2003; Stylianou and Silver 2004)
- Success in solving problems is also related to students’ ability to move flexibly among representations. (Huinker 2013; Stylianou and Silver 2004)
Important Instructional Considerations

- Math drawings and other visual supports are of particular importance for English language learners, learners with special needs, or struggling learners, because they allow more students to participate meaningfully in the mathematical discourse in the classroom (Fuson and Murata 2007).
- Help students to see the connection between the different representations. It isn’t just about showing multiple ways but seeing that the representations are connected to each other mathematically.

Standards for Mathematical Practice

3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.

Student Materials

A variety of manipulatives should be available to students to promote understanding and sense making of the mathematics. (This is not a limited list, but suggestions of tools to have available.)

<table>
<thead>
<tr>
<th>base ten blocks</th>
<th>algebra tiles</th>
<th>connecting cubes</th>
</tr>
</thead>
<tbody>
<tr>
<td>measurement tools</td>
<td>tens frames</td>
<td>100s charts</td>
</tr>
<tr>
<td>pattern blocks</td>
<td>1in. or cm tiles</td>
<td>number lines</td>
</tr>
<tr>
<td>graph paper</td>
<td>calculator</td>
<td>diagrams</td>
</tr>
<tr>
<td>geoboards</td>
<td>tangrams</td>
<td>two color counters</td>
</tr>
<tr>
<td>Cuisenaire rods</td>
<td>rekenreks</td>
<td></td>
</tr>
</tbody>
</table>

Instructional Resources

Eureka Math

FHSD Curriculum Maps

http://map.mathshell.org/materials/index.php

https://www.engageny.org/

https://investigations.terc.edu/

https://www.illustrativemathematics.org/

http://mathsolutions.com/
4. Facilitate meaningful mathematical discourse.

Effective teaching of mathematics
- facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments

<table>
<thead>
<tr>
<th>Teacher Actions</th>
<th>Student Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engage students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.</td>
<td>Present and explain ideas, reasoning, and representations to one another in pairs, small-group, and whole-class discourse.</td>
</tr>
<tr>
<td>Scaffold student approaches and solution strategies for whole-class analysis and discussion.</td>
<td>Listen carefully to and critique the reasoning of peers, use examples to support or counterexamples to refute arguments.</td>
</tr>
<tr>
<td>Facilitate discourse among students by having them justify and explain their reasoning for their answer and approach.</td>
<td>Seek to understand the approaches used by peers by asking clarifying questions, try out others’ strategies, and describe the approaches used by others.</td>
</tr>
<tr>
<td>Ensure progress toward mathematical goals by making explicit connections to student approaches and reasoning.</td>
<td>Identify how approaches to solving a task are the same and how they are different.</td>
</tr>
</tbody>
</table>

Instructional Research Based Strategies

Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics.

Students who learn to articulate and justify their own mathematical ideas, reason through their own and others mathematical explanations, and provide a rationale for their answers develop a deep understanding that is critical to their future success in mathematics and related fields.

Whole class discussions
- anticipating student responses prior to the lesson
- monitoring students work and engagement with the tasks
- selecting particular students’ responses in a specific order to promote a variety of strategies for conceptual understanding
- connecting different students’ responses to key mathematical ideas

Mathematical discourse includes the purposeful exchange of ideas through classroom discussion as well as through other forms of verbal, visual, and written communication.

The discourse gives students opportunities to share ideas and clarify understandings, construct convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other perspectives.

Teachers and students proceed through the levels in shifting from a classroom in which teachers play the leading role to one where they facilitate students’ mathematical thinking.
Important Instructional Considerations

Teacher Role - Students carry the conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for the students to clarify thinking of others.

Questioning - Student to student talk is student initiated. Students ask questions and listen to responses. Many questions ask why and call for justification. Teacher questions may still guide discourse.

Explaining Mathematical Reasoning - Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher.

Mathematical Representations - Students follow and help shape the descriptions of others math thinking through math representations and may suggest edits in others representation.

Building student responsibility within the community - Students believe that they are math leaders and can help shape the thinking of others. They help shape others math thinking in supportive, collegial ways and accept the same support from others.

Standards for Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
8. Look for and express regularity in repeated reasoning.

Instructional Materials Statement

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Student Materials

Materials will be dependent on the lesson that you are building depending on representations, communication tools, justifying, responsive classroom techniques.

Instructional Resources

Classroom Discussions using Math Talk - Marilyn Burns
The First Six Weeks of School-Paula Denton
Number Talks-Sherry Parrish
Generating Math Talk- Marilyn Burns
5. **Pose purposeful questions**

Effective teaching of mathematics

- Uses purposeful questions to assess and advance students' ability to reason and make sense of important mathematical ideas & relationships

<table>
<thead>
<tr>
<th>Teacher Actions</th>
<th>Student Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advance student understanding by asking questions that build on student thinking, but do not take over or funnel ideas to lead students to a desired conclusion.</td>
<td>Expect to be asked to explain, clarify, elaborate on their thinking verbally and in written format.</td>
</tr>
<tr>
<td>Ask questions that go beyond gathering information. The questions should probe students' thinking and require explanation and justification.</td>
<td>Think carefully about how to present their responses to questions clearly, without rushing to respond quickly.</td>
</tr>
<tr>
<td>Ask intentional questions that make the mathematics more visible and accessible for student examination and discussion.</td>
<td>Reflect on and justify their reasoning, not simply provide answers.</td>
</tr>
<tr>
<td>Allow sufficient wait time so that more students can formulate and offer responses.</td>
<td>Listen to, comment on, and question the contributions of their classmates.</td>
</tr>
</tbody>
</table>

**Instructional Research Based Strategies**

Depth of Knowledge Question Stems (Webb, Norman)

Depth of Knowledge

Focusing Pattern of Questioning (Wood, Terry)


RigorRelevance Framework (Daggett, William)


A Framework for types of questions used in mathematics teaching

(e.g. Boaler and Brodie 2004; Chapin and O’Connor 2007)

1. **Gathering Information** - Students recall facts, definitions, or procedures. ex. When you write an equation, what does that equal sign tell you? What is the formula for finding the area of a rectangle? What does the interquartile range indicate for a set of data?

2. **Probing Thinking** - Students explain, elaborate, or clarify their thinking including articulating the steps in solution methods or the completion of a task. ex. As you drew that number line, what decisions did you make so that you could represent 7 fourths on it? Can you show and explain more about how you used a table to find the answer to the Smartphone Plans tasks? It is still not clear how you figured out that 20 was the scale factor, so can you explain it another way?

3. **Making the Mathematics Visible** - Students discuss mathematical structures and make connections among mathematical ideas and relationships. ex. What does your equation have to do with the band concert situation? How does that array relate to multiplication and division? In what ways might the normal distribution apply to this situation?
4. **Encouraging Reflection and Justification** - Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work. ex. How might you prove that 51 is the solution? How do you know that the sum of two odd numbers will always be even? Why does plan A in the Smartphone Plans task start out cheaper but becomes more expensive in the long run?

**Important Instructional Considerations**

- Questioning should encourage students to explain and reflect on their thinking.
- Questioning should allow teachers to discern what students know and adapt lessons to meet varied levels of understanding, help students make important mathematical connections and support students in posing their own questions.
- Questions should attend to what students are thinking, pressing them to communicate their thoughts clearly, and expecting them to reflect on their thoughts and those of their classmates.

**Standards for Mathematical Practice**

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

**Instructional Materials Statement**

FHSD has adopted Eureka Math as the Tier 1 curriculum for grades K-6. Use of this curriculum, Investigations and other standards based resources should be used as advised in the FHSD curriculum maps.

**Student Materials**

Eureka student edition
Journals

**Instructional Resources**

**DOK question stems**

Good Questions for Math Teaching: Why Ask Them and What to Ask, K-6

Number Talks: Helping Children Build Mental Math and Computation Strategies, Updated with Common Core Connections, Grades K-5

Extending Children's Mathematics: Fractions & Decimals: Innovations In Cognitively Guided Instruction

Intentional Talk: How to Structure and Lead Productive Mathematical Discussions

Question Stems to promote 8 mathematical practices

Quick Guide to Questioning in the Classroom
6. **Build procedural and fact fluency from conceptual understanding.**

Effective teaching of mathematics promotes students to be able to

- Build fluency with procedures on a foundation of conceptual understanding
- Become skillful in flexibly using procedures as they solve contextual and mathematical problems

<table>
<thead>
<tr>
<th>Teacher Actions</th>
<th>Student Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acknowledge the importance of both conceptual understanding and procedural fluency but also ensure that the learning of procedures is developed over time, on a strong foundation of understanding and the use of student-generated strategies in solving problems.</td>
<td>Know which procedure is appropriate and most productive in a given situation.</td>
</tr>
<tr>
<td>Provide students opportunities to use their own reasoning strategies and methods for solving problems.</td>
<td>Demonstrate the ability to choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and are able to explain their approaches, and they are able to produce accurate answers efficiently.</td>
</tr>
<tr>
<td>Ask students to discuss and explain why the procedures that they are using work to solve particular problems.</td>
<td>Access procedures that they can use with understanding on a broad range of problems.</td>
</tr>
<tr>
<td>Connect student generated strategies and methods to more efficient procedures as appropriate.</td>
<td>Demonstrate knowledge by practicing on a moderate number of carefully selected problems once they have a strong conceptual foundation and can explain the use of the strategy.</td>
</tr>
<tr>
<td>Use visual models to support students' understanding of general methods.</td>
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<tr>
<td>Provide students with opportunities for continuous practice of procedures.</td>
<td></td>
</tr>
<tr>
<td>Provide students time to practice math facts.</td>
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</tbody>
</table>

**Instructional Research Based Strategies**

“Strategies that Work” Marzano:

Non-linguistic Representations: Ask students to…

- Generate mental images representing content
- Draw pictures or pictographs representing content
- Construct graphic organizers representing content
- Act out content
- Make physical models of content
- Make revisions in their mental images, pictures, pictographs, graphic organizers, and physical models
Important Instructional Considerations

Standards for Mathematical Practice
4. Model with mathematics
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Instructional Materials Statement

FHSD has adopted Eureka math as the Tier 1 curriculum. Use of this curriculum, Investigations and other standards based resources should be used as advised in the FHSD curriculum maps.

Student Materials
-see instructional resources below; student materials are included with these resources

Instructional Resources

Mastering the Basic Math facts in Addition and Subtraction from Susan O’Connell and John SanGiovanni
Mastering the Basic Math facts in Multiplication and Division from Susan O’Connell and John SanGiovanni
Eureka Math - Daily Sprints
Hunt Institute- conceptual understanding.
7. **Support productive struggle in learning mathematics**

Effective teaching of mathematics
- Consistently provides students with opportunities and supports to engage in productive struggle
- Opportunities for delving more deeply into understanding the mathematical ideas
- Able to apply their learning to new problem situations

<table>
<thead>
<tr>
<th>Teacher Actions</th>
<th>Student Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anticipate what students might struggle with during a lesson and being prepared to support them productively through the struggle.</td>
<td>Struggle at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle.</td>
</tr>
<tr>
<td>Give students time to struggle with tasks, and asking questions that scaffold students’ thinking without stepping in to do the work for them.</td>
<td>Ask questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks.</td>
</tr>
<tr>
<td>Help students realize that confusion and errors are a natural part of learning, by facilitating discussion on mistakes, misconceptions, and struggles.</td>
<td>Persevere in solving problems and realizing that it is acceptable to say, “I don’t know how to proceed here,” but it is not acceptable to give up.</td>
</tr>
<tr>
<td>Praise students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.</td>
<td>Help one another without telling their classmates what the answer is or how to solve the problem.</td>
</tr>
</tbody>
</table>

**Instructional Research Based Strategies**
- “Rescuing” students when they face difficulties undermines the efforts of students, lowers the cognitive demands of the task, and deprives the students of opportunities to engage fully in making sense of the mathematics. (Reinhart 2000; Stein et al. 2009)
- “If you are not struggling, you are not learning”. (Carter 2008, p.136)
- Provide students with specific descriptive feedback on their progress related to their making sense of math. (Clarke 2003; Hattie and Timperley 2007)

**Important Instructional Considerations**
- Create a safe environment for learning.
- Consider student struggles and misconceptions.
- Classrooms should embrace productive struggle to necessitate rethinking on the part of both the teacher and student.
- Students question and critique the reasoning of their peers and reflect on their own understanding.
- Students have access to tools that will support their thinking processes.
- Teacher plans for tasks that promote reasoning and problem solving; a solution pathway is not straightforward, but requires some struggle to arrive at the solution.

**Standards for Mathematical Practices:**
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
Instructional Materials Statement

FHSD has adopted Eureka math as the Tier 1 curriculum. Use of this curriculum, Investigations and other standards based resources should be used as advised in the FHSD curriculum maps.

Student Materials:
variety of manipulatives
paper-pencil
access to technology
models of multiple strategies for number operations
access to formulas
steps to problem solving poster/guide
graphic organizers
math journals for reflection/discussion

Instructional Strategies/Resources:
variety of rich problems
real world task problems related to unit
models
gradual release of responsibility
think-pair-share
I do - we do - you do
setting up problem-solving expectations/guide
small group work - establish roles for each group member
assessments - pre/post, exit slips, informal/formal, formative/summative, observations
learning goals stated before, during and after lessons

Resources
Triumph Learning-Productive struggle white paper
Emergentmath.com - The struggle for productive struggle
8. Elicit and use evidence of student thinking

Effective teaching of mathematics
- Uses evidence of student thinking to assess progress toward understanding
- Uses evidence to adjust instruction continually in ways that support and extend learning

<table>
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<tbody>
<tr>
<td>Identify what counts as evidence of student progress toward mathematics learning goals. (Formative Assessment)</td>
<td>Reveal mathematical understanding, reasoning, and methods in written work and classroom discourse.</td>
</tr>
<tr>
<td>Elicit and gather evidence of student understanding at strategic points during instruction.</td>
<td>Reflect on mistakes and misconceptions to improve their mathematical understanding.</td>
</tr>
<tr>
<td>Interpret student thinking to assess mathematical understanding, reasoning and methods.</td>
<td>Ask questions, respond to, and give suggestions to support the learning of their classmates.</td>
</tr>
<tr>
<td>Make in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend.</td>
<td>Assess and monitor their own progress toward mathematics learning goals and identifying areas in which they need to improve.</td>
</tr>
<tr>
<td>Reflect on evidence of student learning to inform the planning of next instructional steps.</td>
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</tbody>
</table>

Instructional Research Based Strategies

Attention to eliciting and using evidence is an essential component of formative assessment. (William 2007a)

“Teachers using assessment for learning continually look for ways in which they can generate evidence of student learning, use this evidence to adapt their instruction to better meet their students’ learning needs.” (Leahy, 2005)

“Identifying indicators of what is important to notice in students’ mathematical thinking, planning for ways to elicit that information, interpreting what the evidence means with respect to students’ learning, and then deciding how to respond on the basis of students’ understanding.” (Jacobs, Lamb, & Philipp 2010; Sleep and Boerst 2010; van Es 2010)

Important Instructional Considerations
- What mathematical strand is being taught
- Developmental considerations
- “Teachers attend to more than just whether an answer is or is not correct.” (Crespo 2000)
- Each lesson needs to include intentional and systematic plans to elicit evidence that will provide information about how student learning is evolving toward the desired goal. (Heritage 2008, p. 6)

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them.
2. Construct viable arguments and critique the reasoning of others.
3. Model with mathematics.
4. Attend to precision.
5. Look for and make use of structure.
6. Look for and express regularity in repeated reasoning.
Instructional Materials Statement

FHSD has adopted Eureka math as the Tier 1 curriculum. Use of this curriculum, Investigations and other standards based resources should be used as advised in the FHSD curriculum maps.

Student Materials
opportunities to say, draw, build, write (paper/pencil, manipulatives)
opportunities to show thinking (including difficulties, mistakes, misconceptions)
discussion time
opportunities for students to make and use connections among mathematical representations

Instructional Resources
assessments (pre, post, informal/formal, summative)
daily check-ins (exit slips, homework, fluency practice)
high-level tasks that require students to explain, represent, and justify mathematical understanding and skills (promoting reasoning and problem solving)

Recommended Reading List
Good Questions for Math Teaching: Why Ask Them and What to Ask, K-6, Peter Sullivan and Pat Lilburn (Copyright © 2002 Math Solutions)
Good Questions for Math Teaching: Why Ask Them and What to Ask, Grades 5-8, Lainie Schuster and Nancy C. Anderson (Copyright © 2005 Math Solutions)
Section 4: Response to Intervention: RtI

The RtI framework is used to meet the needs of all learners in mathematics and across all content areas. By using a systematic approach to know our students as learners, including data analysis and multi-tiered systems of support, we maximize the learning potential for individual students.

RtI is a structured process that schools use to meet the academic, behavioral, and social-emotional needs of all learners. The multi-tiered system of support provides varying layers of intensity and includes strong core curriculum, differentiated instruction, and responsive intervention and enrichment.
**Mathematics Flowchart**  FHSD has created a guiding document for teachers to use as decisions are being made for math intervention with students. This document shows the progression of the use of resources based on data collected from district assessments.

**Subskills and Scaffolds to support grade level standards**
Forest Hills teachers are working on documenting the subskills and scaffolds that build to each math content standard in grades K-12. This work will continue during the 2015-16 school year. Please check back for updates.
Mathematics and the Gifted Student

Forest Hills School District provides a “Math Plus” class to students in grades 4, 5, and 6. This daily advanced math class includes interdisciplinary units connecting math curriculum to Language Arts, Social Studies and Science content. This class is designed for students identified as Superior Cognitive on a state approved standardized intelligence test OR with a cognitive ability of 122 or above and a Math Achievement composite score of 97th percentile or above on the most recently administered nationally normed tests that are on the state approved list.

Ohio’s New Learning Standards provide the foundation for the curriculum within the “Math Plus” class along with additional supplemental materials that challenge high cognitive or high math ability students. Differentiation for these students is critical due to their vastly different academic and social needs.

Instruction within the Math Plus classroom combines rigorous problem solving strategies with compacted daily instruction in math. This instruction includes extensions allowing students to explore areas of interest and talent. Students explore mathematical processes through inquiry to become innovators of mathematical applications. The instruction provides opportunities for students to learn beyond the curriculum in problem-based learning applying their knowledge to real world problems. Students are guided as they explore their passions in 21st century learning. Students also are challenged to discover more about themselves as the social emotional needs of gifted students are nurtured.