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Table of Contents

Section 1: FHSD Philosophy & Policies

FHSD Mathematics Course of Study (board documents)	pages 2-3
FHSD Technology Statement	page 4
FHSD Mathematics Calculator Policy	pages 5-7

Section 2: FHSD Mathematical Practices

Description	page 8
Instructional Guidance by grade level band	pages 9-40
Look-for Tool	pages 41-43
Mathematical Practices Classroom Visuals	pages 44-45

Section 3: FHSD Mathematical Teaching Habits (NCTM	<u>l)</u>
Description	page 46
Mathematical Teaching Habits	pages 47-62

Section 4: Rtl: Response to Intervention

Rtl: Response to Intervention Guidelines	pages 63-64
Skills and Scaffolds	page 64
Gifted	page 65

Section 1: Philosophy and Policies

Math Course of Study, Board Document 2015

Introduction

A team of professional, dedicated and knowledgeable K-12 district educators in the Forest Hills School District developed the Math Course of Study. This document was based on current research in mathematics content, learning theory and instructional practices. The Ohio's New Learning Standards and <u>Principles to Actions: Ensuring Mathematical Success for All</u> were the main resources used to guide the development and content of this document.

While the Ohio Department of Education's Academic Content Standards for School Mathematics was the main source of content, additional sources were used to guide the development of course indicators and objectives, including the College Board (AP Courses), Achieve, Inc. American Diploma Project (ADP), the Ohio Board of Regents Transfer Assurance Guarantee (TAG) criteria, and the Ohio Department of Education Program Models for School Mathematics.

The Mathematics Course of Study is based on academic content standards that form an overarching theme for mathematics study. Six standards flow through mathematics instruction, preschool through high school. The first five are: 1) Number, Number Sense and Operations, 2) Measurement, 3) Geometry and Spatial Sense, 4) Patterns, Functions and Algebra, and 5) Data Analysis and Probability. The sixth Standard is a process standard and includes: 1) Problem Solving, 2) Communication, 3) Connections, 4) Reasoning, and 5) Representation.

The content in the Forest Hills School District Mathematics Course of Study reflects the skills, processes and knowledge we believe students in the Forest Hills School District need to know to be competent, knowledgeable and confident in their understanding of mathematics and in their ability to apply this understanding in future learning experiences.

Philosophy

Learning mathematics involves a variety of skills, processes and understandings. We support a balanced approach to teaching mathematics, which emphasizes both conceptual understanding and procedural fluency and which leads to increased mathematical proficiency for all students in Forest Hills. We believe students must have a comprehension of mathematical concepts, operations and relations as well as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. This is best accomplished through a well-articulated, comprehensive, coherent and consistent mathematics curriculum in all Forest Hills schools, from early childhood years through high school graduation.

Along with conceptual understanding and procedural fluency, students must also have an ability to formulate, represent and solve a variety of mathematical problems. Flexibility, creative thinking, and strategic competence are essential in today's information-driven, quickly changing world. We strive to educate students who not only have the understanding of mathematical concepts and procedural fluency with mathematical skills but who can model mathematical situations appropriately and construct their own learning through carefully planned classroom experiences.

An ability to reason is essential, with a capacity for logical thought, reflection, explanation and justification. Communicating mathematically verbally and in written form is an important step in leading to greater mathematical proficiency, and all students should frequently be asked to reason, communicate, make connections, and represent mathematical ideas.

We as educators will aspire to help all students learn to appreciate mathematics and see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own experience. Research confirms that parent involvement is an important factor in student achievement. We encourage communication with parents about our mathematics teaching and learning and we value their support of the mathematics.

None of the components mentioned above is independent, and all are interwoven and interdependent in the development of mathematical proficiency. The classroom teacher of mathematics is the most important factor in weaving the strands together through daily instruction, differentiating to meet the needs of all students, and collaborating with colleagues to help all students achieve success in mathematics learning.

Technology Statement

As technology becomes essential in our daily activities, the world demands more technological expertise. The vast array of technologies available to teachers should be integrated as a modern tool of instruction to make mathematics more meaningful and accessible for all students.

Forest Hills Calculator Belief Statements and Decision Making Framework:

Calculators should be used when they enhance student learning.

Calculators should be used when the computations become so cumbersome that they are an obstacle to learning higher-level concepts.

Students should be taught how to use calculators in responsible and reasonable ways. Most importantly, students should be able to accurately interpret and understand the answers provided by a calculator.

Students should be taught when to use a calculator and when mental computing is more effective or appropriate. Choosing the right "tool" is part of an effective problem-solving process.

Calculators are an essential tool:

- for the discovery of generalizations or patterns within mathematics
- when making sense of mathematics
- for communicating mathematical thinking

Forest Hills calculator belief statements are based upon:

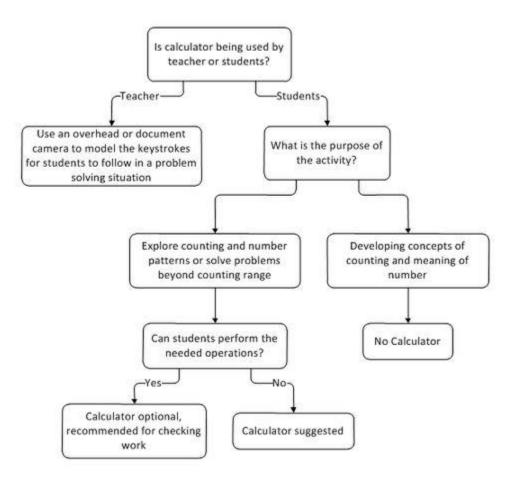
- The Ohio Department of Education Assessment Guidelines
- Research highlighted in <u>Principles to Actions: Ensuring Mathematical Success for All. p. 78-83:</u> "Despite popular belief, use of technology does not inhibit students' learning of mathematics. The idea that it does is particularly prevalent regarding the use of calculators. However, after conducting a comprehensive literature review, Ronau and others (2011, p.1) concluded the following:

In general, we found that the body of research consistently shows that the use of calculators in the teaching and learning of mathematics does not contribute to any negative outcomes for skill development or procedural proficiency, but instead enhances the understanding of mathematics concepts and student orientation toward mathematics."

• Standards for Mathematical Practice #5:

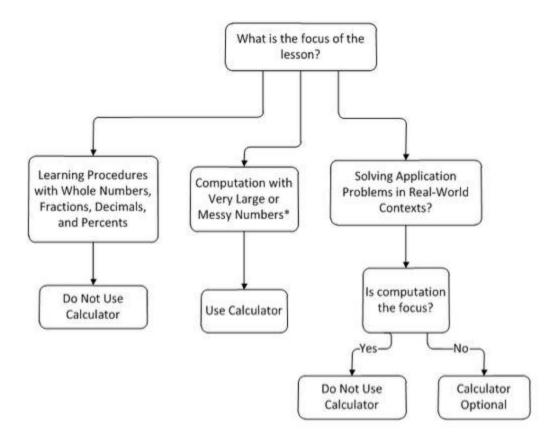
Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts. The following charts were developed by the West Virginia Department of Education for instructional decision making regarding the use of calculators. The Forest Hills Course of Study Committee endorses their use in lesson design, instructional planning, and assessment.



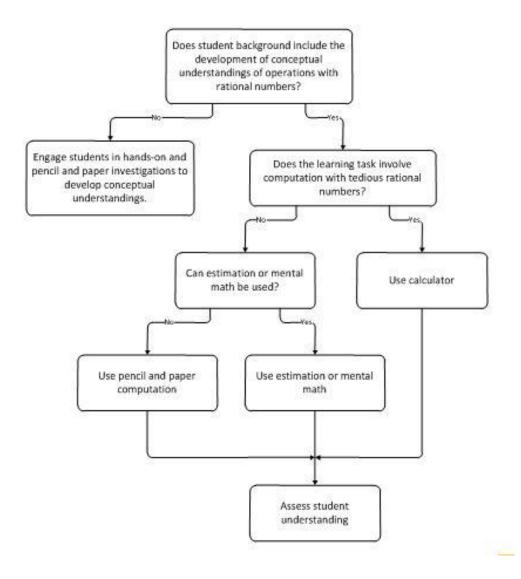
Calculator Usage in the Primary Grades

Calculator Usage in the Intermediate Grades



*Students with special needs may require modifications including using the calculator when performing multi-digit computation

Operations with Rational Numbers



Section 2: FHSD Mathematical Practices

The Standards for Mathematical Practices describe ways student practitioners are developed as they engage with content and develop a balanced combination of procedures, processes, and understanding.

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

On the following pages you will find a snapshot of each practice by grade band. It includes a detailed definition, kid friendly "I can" statements, examples of teacher's role and student's role to guide a teacher's instructional planning and integrate with the content standards. We use these practices to teach Ohio's New Learning Standards.

- Grades K-1 Math Practices
- Grades 2-3 Math Practices
- Grades 4-6 Math Practices

The Forest Hills School District Mathematical Practices Look-For Tool was created through a K-12 collaborative team while analyzing the practices both in the classroom and using current research. The tool was designed to be used by teachers to grow professionally through reflection of their practices. An administrator or colleague can provide a teacher feedback using this tool constructively.

MP1 Makes sense of problems and perseveres in solving them

Definition Students understand and look for a way to solve the problem. They can explain their process, monitor their work, and prove their solutions. "Do I understand the problem?" "Did I persevere?" "Does my solution make sense?"	 Student Friendly I can explain/understand what the problem is asking. I can make a plan. I can get unstuck when I am stuck. I can change my plan if it isn't working. I monitor my work. I can prove that my answer makes sense. I can clearly explain my answers.
their work, and prove their solutions. "Do I understand the problem?" "Did I persevere?"	 I can make a plan. I can get unstuck when I am stuck. I can change my plan if it isn't working. I monitor my work. I can prove that my answer makes sense.

Examples:

Teachers	Students
 provide open-ended rich problems ask probing questions model solving multi-step problems scaffold instruction provide a safe environment for learning from mistakes and collaborating together model how to find key information model a variety of strategies allow time for student-led discussions probe for a variety of different strategies 	 share and discuss strategies kids work in groups collaboratively persevere through frustrations use a variety of strategies and methods to solve a problem monitor their problem solving process prove their solutions in more than one way re-read and retell the problem in their own words find key information check to see if their answer is reasonable

Teachers	Students
 teach isolated skills not connected to other learning do not access or build on prior knowledge do not create a safe learning environment for kids to make mistakes. ask closed questions versus open-ended 	 do not explain solutions or problem solving process quit due to frustration do not make connections or use their prior knowledge and experiences do not evaluate if their solution is a reasonable answer

MP2 Reason abstractly and quantitatively

Definition Students can visualize problems. They can share their thinking with others to solve problems and explain their solution. Students can create multiple representations to explain their thinking (using pictures, words, number sentences, tables, graphs).	 Student Friendly I can make sense of the problem. I can use numbers, pictures, words, equations to show my thinking. I can prove that my answer makes sense.
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Examples

Teachers	Students
 develop opportunities for problem solving strategies model how to use problem solving strategies provide real world situations value feedback given for invented strategies/representations used teach students to focus on steps taken to solve the problem, not necessarily focusing on the answer 	 create multiple representations to explain their thinking (pictures, words, number sentences, tables, & graphs) learn to self-check - Does my answer make sense? (i.e. 5-3 ≠ 8, because my answer should be less than 5) visualize problems

Teachers	Students
 do not model or allow time to practice provide rigid teacher centered environment limit variety of questions and provide no real-world connection do not provide a safe environment for students to share invented strategies/representations have a lot of emphasis on the answer only teach by teacher led discussion 	 lack the understanding that math relates to and is used in everyday life use paper/pencil tasks only are unaware of strategies and relationships are confused with tasks and strategies are unable to explain their thinking give unreasonable answers

MP 3 Construct viable arguments and critique the reasoning of others

Definition Students can explain to others how they solved the problem. Students can listen and respond and critique the thinking and reasoning of others.	 Student Friendly I can decide if my peers' answers make sense. I can construct, explain and prove my answer.
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Examples

Teachers	Students
 create a safe environment for risk taking and critiquing with respect use feedback to model desired student discourse provide complex, rigorous tasks that foster deep thinking plan effective questions, student grouping and time for student discourse probe students to explain their thinking model how to talk about and share solutions model how to be a respectful participant (speaker or listener) when others are sharing encourage the use of mathematical 	 ask questions for clarification of another student's thinking analyze others arguments use examples and counterexamples observe mathematical patterns and make reasonable arguments use objects, drawings, diagrams, and actions to support their thinking use mathematical vocabulary

Teachers	Students
 do not promote discourse or conversation among students through feedback lead discourse and not students do not foster risk taking and respect for the thinking of others in their classroom ask only simple questions attend only to the answer not the process use language that is not precise (not mathematical) 	 accept answers of others without questioning respond inappropriately to the thinking of others are unaware of mathematical patterns and unable to support their thinking use only one method to support their thinking use vague and imprecise mathematical language

MP4 Model with Mathematics

Definition Students can find key information and use it to make a representation to draw conclusions and determine if it makes sense.	 Student Friendly I can use objects or pictures to show my thinking. I can use equations to represent my thinking.
"Does my model/representation match the problem?" "Does my answer make sense?"	

Examples

Teachers	Students
 allow time for the process to take place model the process not the product make appropriate tools available create a safe environment where risk taken is valued provide meaningful, real-world, authentic, performance-based tasks allow for discourse and investigation 	 realize they use mathematics to solve real-world situations use number relationships to draw conclusions (if 5+5=10 then 6+4=10) determine if the solution makes sense and make corrections if needed understand the tasks being asked

Teachers	Students
 provide lack of modeling or time to practice limit tools use rigid teacher centered environment use limited questions and provide no real- world connection only teach through teacher led discussion 	 lack understanding that math relates and is used in everyday life use paper/pencil tasks only are unaware of strategies and relationships are confused with the tasks and strategies are unable to explain their thinking

MP5 Use Appropriate Tools Strategically

Definition Students can choose the appropriate tools to solve the given problem. They can use the appropriate tool for the situation at hand.	 Student Friendly I can choose the appropriate math tool to solve a problem. I can use math tools to show how I solved a problem. I can use math tools to help me solve a problem.
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Examples

Teachers	Students
 model use of appropriate tools provide opportunities for students to discover which tool is appropriate and why compare/contrast the effectiveness of the tools have a variety of tools available and encourage use of these tools (number lines, pictures, counting jars, tiles, number writing practices) facilitate discussion with student experiences using tools (the why or why not of each tool) 	 choose appropriate tools to solve a given problem recognize the usefulness and limitations of different tools (number lines, pictures, counting jars, tiles, number writing practices)

Teachers	Students
 only provide modeling for one tool provide limited time for discovery of tools provide little or no discussion regarding efficiency of tools do not provide instruction on how to use each tool (tape measurer) do not refer to manipulatives as tools (instead of toys) 	 use math tools as toys do not pick the tool that is most appropriate do not verbalize their rationale for use of the tool chosen

MP6 Attend to Precision

Definition Student can understand and use math vocabulary. Student can solve problems accurately and talk about their thinking. They can work carefully and check their work.	 Student Friendly I can work carefully and check my work. I can understand and use math vocabulary. I can solve problems accurately and talk about my thinking.
"Is my answer correct?" "How can I prove it?"	

Examples

Teachers	Students
 use mathematical language, and or charts, labeling shelves with pictures and words link new vocabulary to common vocabulary model using manipulatives, pictures, symbols or verbally explain/think aloud how to check work ask open ended questions encourage fruitful mistakes are open to divergent thinking value the process, not just the product 	 use math language show thinking with manipulatives, pictures, symbols, verbally double check work show thinking in more than one way partner chat to think about their work work toward efficient thinking

Teachers	Students
 tell how to solve the problem show only one way do not question students reasoning teach skills in isolation do not provide opportunity for classroom discourse 	 only try one strategy only try one tool show only product, not the process work in isolation

MP 7 Look for and make use of structure

Definition Students use prior knowledge (what they already know) to solve new problems. They can break down complex problems into simpler, more manageable chunks. Students can recognize and understand the patterns they see in problem situations.	 Student Friendly I can find patterns in numbers. I can use patterns in numbers to solve problems. I can take apart and put numbers back together. I can break down problems into easier parts.
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Examples

Teachers	Students
 use open ended questioning are quiet and allow time for student discussion foster persistence/stamina in problem solving by modeling and practice provide students with tasks where they can look for structures and patterns 	 look for and identify patterns in numbers use skills previously learned to solve new problems break down complex problems into simpler tasks understand how others solved the problem differently

Teachers	Students
 lead lessons without student discussion or questioning provide one type of problem repeatedly explain without modeling provide clarification and explanations quickly 	 do not see the patterns in numbers use a skill in isolation only do not transfer a skill to a new situation attempt to solve complex problem all at once do not understand a different explanation from your own

MP8 Look for and express regularity in repeated reasoning

Definition Students see mathematical patterns when solving problems. Students are able to make generalizations based on the patterns they see.	 Student Friendly I can solve problems by looking for patterns that repeat and use those patterns to solve other problems. I can make generalizations based on patterns.
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Examples

Teachers	Students
 provide rich and varied tasks that allow students to generalize relationships and methods, and build on prior mathematical knowledge provide adequate time for exploration, dialogue and reflection, peer collaboration ask deliberate questions that enable students to reflect on their own thinking periodically throughout the process create strategic and intentional check-in points during student work time 	 identify patterns and make generalizations continually evaluate reasonableness of intermediate results maintain oversight of the process search for, identify, and use short-cuts

Teachers	Students
 provide one type of problem provide limited problems for exploration isolate individual skills only assess at the end of unit lead lessons with no discussion 	 quickly answer without justification are unable to make connections view tasks in insolation do not see any repeated patterns in their work

MP1 Makes sense of problems and perseveres in solving them

Definition Students can explain to themselves the meaning of a problem and look for entry points to a solution They can plan a solution pathway- not jump into a solution attempt. Students can use prior knowledge to plan and solve, continually asking, "Does it make sense?" They will monitor and evaluate progress and change course if necessary. Students will check work using different methods/strategies and explain relationships between equations, graphs, diagrams, etc.	 Student Friendly I can explain/understand what the problem is asking. I can make a plan. I can make a plan using what I already know. I can check to be sure that my answer makes sense. I can change my plan if it isn't working out. I can solve my problem in different ways. I can use representations to support my solution.
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Examples

Teachers	Students
 provide open-ended, rich problems ask probing questions provide opportunities for reflection (i.e. math journals) provide rubrics for assessment and self-reflection use manipulatives and technology to support strategies display and discuss student work samples to show a variety of strategies provide opportunities for student discourse and collaboration (i.e. Think-Pair-Share) promote a safe environment to learn where students feel comfortable taking risks 	 thoughtfully read information given - What do I notice? highlight key words, numbers, and phrases within the question to help make sense of the problem ask themselves if they need other information to solve choose efficient strategies (plan) to solve draw pictures, use representations, use manipulatives to create a plan reflect to see the reasonableness of answers collaborate by talking with peers about math communicate their thinking orally and in writing

Teachers	Students
 use limited math and language vocabulary have students working only independently teach a single algorithm as the "way" to solve always have a quiet classroom don't provide time to share 	 work with homework and classwork in a "drill and kill" fashion do not use math manipulatives to demonstrate understanding don't share their thinking give only one strategy to solve a problem are required to solve problems a certain way

MP2 Reason abstractly and quantitatively

Definition Students make sense of the quantities and use reasoning skills to understand relationship to the problem at hand. Students reason and recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. Students can solve problems in context. They can also pull out of a context to solve or represent with a number pattern.	 Student Friendly I can make sense of the problem. I can write an equation that matches the problem. I can use numbers, pictures, words, equations to show my thinking. I can prove that my answer makes sense.
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Examples

Teachers	Students
 encourage and model reasoning and making sense of problems use concrete models to foster habits of reasoning and create representations focus on the meaning of quantities - pairing visual images to representations and symbols connect meaning of quantities to computations and properties of operations 	 develop habits of reasoning and creating of representations of the problem at hand consider the units involved attend to the meaning of quantities make sense of quantities and their relationship in problem solving understand the meaning of quantities and are flexible in the use of operations and their properties THINK - Engage - What makes sense?

Teachers	Students
 use a lot of drill and practice worksheets focus on mostly on memorization rather	 rush through problems with little time spent
than reasoning skills don't allow talking promote the philosophy, "just do the work	in discussion or questioning don't make sense of quantities don't try to contextualize or decontextualize
quietly" have an attitude of "my way or the highway"	to solve problems do not represent math symbolically

MP3 Construct viable arguments and critique the reasoning of others

Definition Students need to articulate their reasoning and identify what works and what doesn't in the reasoning of others. Their conversations should show understanding of the concept to build a logical progression of statements, as well as recognizing and using counterexamples. Students should justify their conclusions, communicate them to others and respond to the arguments of others.	 Student Friendly I can explain my thinking and support it with math words, symbols and visuals. I can listen to the reasoning of others and decide if it is reasonable. I can apply the reasoning of others to a math problem.
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Examples

Teachers	Students
 create a safe environment for risk-taking and critique with respect model desired student discourse provide complex, rigorous tasks that foster understanding/opportunities for discussion plan and use effective questions use effective grouping of students provide time for student discourse 	 ask questions to clarify misconceptions use examples and nonexamples make sense in their reasoning and explain their thinking clearly compare two plausible arguments, distinguish correct reasoning from flawed reasoning, and explain why an argument is flawed

Teachers	Students
 is seen as the sage on the stage dictates one way to find a solution 	 learn and work silently focus on one person's work and one way to solve a problem hurt others feelings due to personal attack argue inappropriately about work samples

MP4 Model with Mathematics

Definition Students can apply the mathematics they know to solve real world problems. This includes writing an equation to describe and solve a situation.	 Student Friendly I can use geometric figures, pictures, or physical objects or diagrams such as a number line, a table or graph to represent a real world problem. I can show my work in many ways. I can use expressions or equations to represent my thinking.
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Examples

Teachers	Students
 allow time for the process to take place (model, make graphs) model desired behaviors (think alouds) and thought processes (questioning, revision, reflection/written) make appropriate tools available for students to select from create an emotionally safe environment where risk taking is valued provide meaningful, real world, authentic, performance-based tasks (non-traditional word problems) encourage and model discourse encouraging students to investigate their own mathematical curiosities 	 realize they use mathematics (numbers and symbols) to solve/work out real-life situations analyze relationships to draw conclusions interpret mathematical results in context show evidence that they can use their mathematical results to think about a problem and determine if the results are reasonable; if not, go back and look for more information make sense of the mathematics simplify a complicated situation identify important quantities

Teachers	Students
 limit time for students to process provide a lack of opportunity for discourse limit or allow no tools to be available for students to demonstrate understanding do not model demonstrate lack of planning provide students with low level traditional word problems 	 work completely independent of others often do not provide an explanation with their answer do not use models or equations to represent their thinking write simple answers do not see mathematics as a tool to solve real world problems

MP5 Use Appropriate Tools Strategically

Definition Students consider the available tools when solving a mathematical problem. Tools should be a variety of manipulatives and technology. Students are sufficiently familiar with tools appropriate for the	 Student Friendly I can choose and use the most appropriate tool for solving a problem. I am familiar with lots of different tools I can use to solve math problems.
sufficiently familiar with tools appropriate for the task.	use to solve math problems.

Examples

Teachers	Students
 maintain knowledge of appropriate tools effectively model use of the tools available, their benefits and limitations model a situation where the decision needs to be made as to which tool should be used compare/contrast effectiveness of tools make available and encourage use of a variety of tools 	 choose the appropriate tool to solve a given problem and deepen their conceptual understanding (paper/pencil, ruler, base 10 blocks, compass, protractor) choose the appropriate technological tool to solve a given problem and deepen their conceptual understanding (e.g., spreadsheet, geometry, software, calculator, web 2.0 tools) compare the efficiency of different tools recognize the usefulness and limitations of different tools

Teachers	Students
 do not show awareness of effective tools limit the availability of tools to what they would choose 	 relies solely on paper and pencil as a tool relies solely on mental math when the task warrants more

MP6 Attend to Precision

Definition Precise clear communication of mathematical process and reasoning; includes work, algorithms, language, etc.	 Student Friendly I can work carefully and check my work. I can understand and use the correct math vocabulary. I can solve problems accurately and talk or write about my thinking I can use appropriate labels with my answers.

Examples

Teachers	Students
 understand precise definitions and context use appropriate vocabulary throughout instruction correct miscues clarify vocabulary display appropriate math language 	 communicate with precision calculate efficiently and accurately explain mathematical reasoning using precise language, tools, pictures, labels, etc. organize thinking ask purposeful questions check answers for accuracy work towards clarity in discussions by carefully formulating an explanation

Teachers	Students
 do not understand the precise meaning of important mathematical vocabulary rush thinking limit resources do not support and provide opportunities for student engagement 	 have sloppy/incomplete labeling show inaccurate computation overuse the words "it" or "thing" use lots of everyday language lack explanation

MP7 Look for and make use of structure

Definition Students look closely to discern a pattern or structure. Examples would be commutative property, associative property, decomposing a number with place value within addition and subtraction strategies, mentally adding/subtracting 10 and 100.	 Student Friendly I can identify patterns within a problem and use efficient strategies to solve. I can see and understand how numbers and shapes are organized and put together as parts and wholes. I can break down complex problems into smaller chunks to solve accurately.
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Examples

Teachers	Students
 facilitate learning by using open-ended questioning to assist students in exploration carefully select tasks to promote opportunities to look for and identify patterns guide students to use patterns to make generalizations provide time for student discussion and processing (i.e. wait time) foster persistence and stamina in problem solving through practice and modeling 	 look closely to discern patterns and structure recognize, reflect on and interpret patterns and structures using number grid, number line, number bonds, place value chart and ten-frames use previously learned skills and strategies to solve new problems and tasks decompose numbers into more workable numbers such as hundreds, tens and ones decompose shapes into more workable shapes

Teachers	Students
 do not provide mathematical structures to help students understand the mathematics use a lot of low level problem types are not thoughtful about the types of relevant practices provided to students to identify patterns teach through teacher led discussion most of the time, tell students what the relationships are instead of discovery and inquiry limit student participation 	 are not engaged or involved do not make connections or use previously learned strategies do not use mathematical patterns or structures to make sense of the problem cannot apply mathematical methods/algorithms to not traditional problems

MP8 Look for and express regularity in repeated reasoning

Definition Students notice if calculations are repeated, and look both for general methods and patterns to use as shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process while attending to details. They continually evaluate the reasonableness of the intermediate results.	 Student Friendly I can solve problems by looking for patterns that repeat and rules that can apply to other problems. I look carefully at the entire problem and pay attention to details. I repeatedly ask myself, "Does my answer make sense?" I am looking for patterns to help me solve my work more efficiently.
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Examples

Teachers	Students
 provide rich and varied tasks that allow students to generalize relationships and methods that will build on prior mathematical knowledge provide adequate time for exploration, dialogue and reflection, peer collaboration ask deliberate questions that enable students to reflect on their own thinking periodically throughout the process create strategic and intentional check in points during student work time 	 continually evaluate the reasonableness of results search for generalizations to identify and use patterns as short-cuts notice repetition and regularity of patterns, for example odd and even patterns or patterns of operations attend to details

Teachers	Students
 provide only repetitive skill and drill practice that do not promote recognition of patterns for students provide only knowledge-based "recall" questioning do not expect students to justify their thinking with reasoning or generalizations do not provide opportunities for students to make connections between addition and subtraction as different ways to solve the same problem do not provide opportunities for students to make connections between multiplication and division as different ways to solve the same problem 	 give limited response do not apply generalizations or patterns in their responses do not make connections to patterns and relationships do not see the relationship between addition and subtraction or multiplication and division

MP1 Makes sense of problems and perseveres in solving them

Definition Students will interpret and analyze to find the meaning of the problem/make sense of the problem. They will create a plan to solve the problem, monitor their progress and change the approach if necessary.	 Student Friendly I can explain/understand what the problem is asking. I can make a plan. I can get unstuck when I am stuck. I can change my plan if it isn't working out. I can monitor my work. I can prove that my answer makes sense. I can show that my representations in my work support my solution.
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Examples

Teachers	Students
 provide open ended and rich problems ask probing questions model multiple problem-solving strategies through <i>think aloud</i> promote and value disclosure and collaboration examine student responses (correct or incorrect) for understanding and multiple representations promote a safe environment to learn in view mistakes as learning opportunities differentiate based on student needs encourage self-monitoring 	 thoughtfully read information given - What do I notice? underline the question being asked make sense of the problem by double underlining key direction words, circling information/numbers needed to solve highlight key math vocabulary or terms in the problem reread thoughtfully ask themselves if they need other information to solve the problem choose efficient strategies (Plan) to solve draw math pictures, use representations, manipulatives to create a plan check to see - is my answer reasonable? does it make sense of the question asked? change plan if needed

Teachers	Students
 focus on telling the students "how to solve" rather than building problem solving strategies simplify language so that students don't have to think to solve the problem use limited mathematical language 	 work alone have not developed a clear path for solving problem are more interested in the "how to" than in understanding the bigger concepts and understanding why do not show representations of their understandings

MP2 Reason abstractly and quantitatively

Definitions

Students understand what the numbers represent. They show how to represent the problems using symbols and numbers. Students can develop more than one strategy or solution to a problem. Learners understand the relationships between problem scenarios and mathematical representation.

Student Friendly

- I can make sense of the problem.
- I can show my process to get a solution.
- I can check my answer to see if it makes sense.
- I can use numbers, words and reasoning to help me make sense of problems.
- I understand what my solution means.

Examples

Teachers	Students
 develop opportunities for and model problem solving strategies provide real world situations makes connections between content areas value invented strategies and representations give less emphasis to the answer help their learners understand the relationships between problem scenarios and mathematical representation 	 estimate first to make more sense of the answer make sure the answer is reasonable create multiple ways to represent their problem solving (tables, pictures, words, symbols) represent the unit

Teachers	Students
 expect no explanation with the answer do not connect math to the real world or other content areas 	 provide answers that are not labeled or do not make sense do not pull out numbers/information to effectively to solve the problem

MP3 Construct viable arguments and critique the reasoning of others

Definitions

Students engage in active mathematical discourse, this might involve having students explain and discuss their thinking processes aloud or signaling agreement/disagreement with a hand signal. A teacher might post multiple approaches to a problem and ask students to identify plausible rationales for each approach.

Student Friendly

- I can make conjectures and critique of the mathematical thinking of others.
- I can construct, justify and communicate arguments by...
 - 1. considering context
 - 2. using examples and non-examples
 - 3. using objects, drawings, diagrams and actions
- I can critique the reasoning of others by...
 - 1. listening
 - 2. comparing arguments
 - 3. identifying flawed logic
 - 4. asking questions to clarify or improve arguments

Examples

Teachers	Students
 create a safe environment for risk-taking and critiquing with respect use feedback to model desired student discourse provide complex, rigorous tasks that foster deep thinking plan effective questions, student grouping and time for student discourse probe students' thinking to gain insight into their understanding 	 ask questions reason inductively and make plausible arguments use examples and counterexamples to prove their understanding or arguments analyze others arguments

Teachers	Students
 focus on just the answer teach procedural methods without understanding use a lot of "drill and kill" problem practice provide feedback that does not provide any room for further discourse (i.e. right or wrong) 	 don't generate the questions completely quiet classroom memorize procedures without reasoning do not share thinking between themselves in whole group and small group settings

MP4 Model with mathematics

Definition Students construct visual evidence using symbolic and graphical representations. They use pictures,	 Student Friendly I can use geometric figures, pictures, or physical objects or diagrams such as a
numbers and words appropriate to the real world context of the problem.	 number line, tape diagram, table or graph to represent the problem. I can show my work in multiple ways. I can use expressions or equations to represent my thinking.

Examples

Teachers	Students
 provide open ended questions/problems that are meaningful, real world, authentic, performance-based tasks demonstrate the process of modeling share students' examples allow time for the process to take place (model, make graphs) provide many opportunities to connect and explain the different relationship between the different representations 	 represent work in pictures (models), numbers and words need many opportunities to connect and explain the different relationship between the different representations show evidence that they can use their mathematical results to think about a problem and determine if the results are reasonable; if not, go back and look for more information

Teachers	Students
 provide lower level multiple choice questions with no room to solve problems focus instruction on procedural method 	 perform using algorithms only use limited representation to explain reasoning can't explain, justify or model

MP5 Use appropriate tools strategically

Definition Students identify and use tools to scaffold their learning and increase understanding of concepts, problem-solving strategies, and to further exploration of possible solutions. They make good decisions about the appropriateness of the specific tool or tools to be used.	 Student Friendly I can choose the most appropriate tool for each given problem. I can use math tools to show how I solved a problem. I can use math tools to help me solve a problem. I know HOW to use math tools.
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Examples

Teachers	Students
 model use of tools - ex. when is an appropriate time to use a calculator; when is a ruler needed? use square tiles to model arrays build understanding of concepts through the use of models and manipulatives - moving from concrete to representation to abstract model use of resources - how to use the mathematical dictionaries, online tools - to help develop math vocabulary and broaden understanding of math concepts identify and model use of online tools 	 use manipulatives to build representations and problem-solving strategies identify math vocabulary - use math resources to determine meaning of unknown math terms will choose the appropriate tool to help them problem solve in an efficient/strategic manner strategically and thoughtfully use tools when appropriate in problem solving situation demonstrate when and how to use math tools

Teachers	Students
 use the same tool over and over and over again without utilizing a variety of different tools and/or teaching methods allowing the use of prohibited tools (unless accommodations are needed) do not allow students to make sense of needed tools based on the mathematics do not have tools available for easy access for students to use when they need to use them 	 repeated use of tool without understanding use the tool when other methods are more efficient and appropriate (99-97 does not need a calculator) do not make sense of the tool and if it is the best tool for the mathematics at hand

MP6 Attend to precision

Definition	Student Friendly
Students calculate accurately and efficiently. They use clear and concise communication, written & oral to explain their understanding and thinking. Students use correct mathematical vocabulary and symbols to communicate their thinking.	 I can work carefully and check my work. I can understand and use math vocabulary. I can solve problems accurately and talk and write about my strategies and solutions.

Examples

Teachers	Students
 create a classroom environment that is safe for risk taking, communication and evaluation of each other's thinking ask probing questions that require students to analyze their thinking and critically evaluate their reasoning model mathematical language, model use of resources value the process, not just the product model/encourage think aloud & double checking of work 	 represent their work to match what the problem is asking talk to one another about their mathematical thinking, share ideas with one another, ask questions of one another demonstrate mathematical fluency by efficiently, accurately and fluently adding, subtracting, multiplying and dividing use math language/vocabulary appropriately use resources to support understanding of math vocabulary

Teachers	Students
 do not provide problems that allow for extended responses focus on only one path to get an answer do not understand the precise meaning of important mathematical vocabulary rush thinking limit resources do not provide opportunity for student engagement 	 work in isolation are not discussing strategies and solutions with peers are sloppy with their representations and work is incomplete missing labeling that shows understanding have inaccurate computation overuse of the words "it" and "thing" lots of everyday language lack explanation in their work

MP7 Look for and make use of structure

Definition Students look for, interpret and identify patterns and structures. They see complicated things as single objects or being composed of several objects. Students make connections to skills and strategies previously learned to solve new problems/tasks independently and with peers.	 Student Friendly I can notice when calculations are repeated. Then, I can find more efficient methods. I can see and understand how numbers and shapes are organized and put together as parts and wholes. I can take complex problems into simpler, more manageable chunks. I can use the structure of mathematics to make sense of my thinking.
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Examples

Teachers	Students
 understand the properties of operations so they can help students make connections show connections between types of numbers; fractions, decimals provide rich problems to look for repeated reasoning and connections 	 show how patterns emerge in rich problems use repeated patterns to find a more efficient way of solving use repeated patterns to show understanding of mathematics

Teachers	Students
 do not provide mathematical structures to help students understand the mathematics use a lot of low level problem types are not thoughtful about the types of relevant practice provided to students to identify patterns teach through teacher led discussion most of the time, tell students what the relationships are instead of discovery and inquiry limit student participation 	 do not use mathematical patterns or structures to make sense of the problem are not able to make connections or use previous strategies. cannot apply mathematical methods/algorithms to non-traditional problems are sitting for long time periods are not engaged or show little involvement

INP8 Look for and express regularity in repeated reasoning	
Definitions	Student Friendly
Students notice if calculations are repeated and look for general methods and shortcuts. They maintain oversight of the process while attending to details. Students continually evaluate the reasonableness of immediate results.	 I can solve problems by looking for patterns that repeat and rules that can apply to other problems. I look carefully at the entire problem and pay attention to details. I repeatedly ask myself, "Does my answer make sense?" I am looking for patterns to help me solve

MP8 Look for and express regularity in repeated reasoning

Examples

Teachers	Students
 provide rich and varied tasks that allow students to generalize relationships and methods, and build prior mathematical knowledge provide adequate time for exploration, dialogue and reflection, peer collaboration ask deliberate questions that enable students to reflect on their own thinking periodically throughout the process create strategic and intentional check in points during student work time 	 ask: is there a pattern here? wonder: how can I generalize this pattern? search for and recognize there is a predictable pattern that will help me in my mathematical understanding ask: is there a shortcut based on a repeated pattern, will it provide for more efficiency and accuracy?

Non-examples

Teachers	Students
 do not probe students' to make observations about patterns provide repetitive skill and drill practice only ask knowledge-based "recall" questioning 	 give limited response do not apply generalizations or patterns in their responses do not make connections to patterns and relationships do not see the relationships in number or patterns

my work more efficiently.

MP1 Make sense of problems and persevere in solving them.

Definition Students engage in understanding and solving complex problems while looking for an entry point. They consider all aspects of the problem and create a solution path before jumping in. Students monitor progress and change pathways if necessary as they continually ask if their answer makes sense. They can use alternate methods to check their answer and understand alternate methods. Overall, students continually work towards a solution, overcoming mistakes or incorrect pathway attempts.	 Student Friendly I can understand a complex problem by breaking it down. I can identify the goal of a complex problem and work with the given information. I can identify and correct mistakes. I can change my approach if necessary. I know multiple methods to solve a problem. I can understand methods presented by others. I can explain that my answer makes sense. I can work without giving up.
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Examples

Teachers	Students
 Teachers will model steps and provide scaffolds to complete complex problems. Teachers will monitor student progress and frustration levels. Teachers will assist students in finding entry points and identify the goal of a problem. Teachers will provide different levels of complex problems. Teachers will facilitate failure as a part of problem solving and show it will enhance student learning and growth. Teachers will provide an environment that is safe and encouraging to problem solving. 	 Students can develop a starting point and identify the consecutive steps necessary to arrive at a solution. Students will allow frustration and be patient when they have reached a brick wall. Students will look for errors in past conclusions when they cannot move forward in a problem. Students can write or verbally explain why they took the necessary steps to solve the problem. Students will not be hindered by failure. Students may utilize multiple approaches when solving the problem.

Teachers	Students
 Teacher gives the student the answer. Teacher provides only simple problems. Teacher does not model or scaffold complex problems. Teacher does not allow for multiple approaches in solving a problem. Teacher does not communicate to students that failure can be an opportunity for growth. Teacher does not identify frustrated students. 	 Student gives up after one try. Student does not go back to use strategies shown or modeled. Student does not use prior and current knowledge to solve. Student becomes frustrated and gives up. Student does not or cannot understand alternate methods. Students believe they cannot solve the problem.

MP #2 Reason abstractly and quantitatively

Definition Students can create multiple representations, connect them together and make sense of the relationship between them. For example, write an equation and create a graph to represent a set of data. Students can make sense of these quantitative relationships in and out of context. Students can identify and use units to help explain what an answer means in the context of a situation. Students know and use different properties of operations and objects flexibly.	 Student Friendly I can create and connect multiple representations and make sense of relationships between them. tables pictures words symbols graphs I can make sense of numbers and symbols mathematically and in real-world contexts. I can describe what an answer means in the context of a situation.
	 I can reason through my process and decide if my answer makes sense.

Examples

Teachers	Students
 Teachers use real world problems to teach concepts. Teachers collaborate with other content area teachers to show application of mathematical topics. Teachers provide opportunities for students to explain answers with multiple representations. Teachers provide opportunities for students to estimate and critique their answers (justify why their answer is reasonable). 	 Students support their answer using mathematical reasoning (and can do so using multiple representations). Students apply prior learning to new real world situations. Students evaluate an answer to determine whether or not it is reasonable. Students complete multiple steps in a solution process, if necessary.

Teachers	Students
 Teacher focuses on the final product (answer). Teacher provides only skill practice, not real world problems. Teacher does not connect content to real world or other applications. 	 Student gives only the answer without reasoning or units. Student can only apply skill in one situation. Student can only represent data in one way. Student cannot decide whether or not an answer is reasonable. Student uses inappropriate information when solving word problems.

MP3 Construct viable arguments and critique the reasoning of others.

Definition Students dig deeply into situations to conjecture, analyze and explore the truth of their arguments/reasoning. Students engage in discussions centered on communicating their thinking, listening to others and responding to critiques or questions. Students compare arguments and establish if an argument is flawed and explain why with evidence. Students consider contexts, special cases and logic progression.	 Student Friendly I can construct a solid argument that: explains my thinking to the group. makes use of mathematical vocabulary. justifies conclusions. provides mathematical evidence. I can determine if another person's argument is mathematically sound and justify why I agree/disagree with it. I can compare the effectiveness of two arguments. I can ask useful questions to clarify or improve upon my or another's reasoning.
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Examples

Teachers	Students
 Teacher provides a safe environment for risk-taking. Teacher models desired discourse - including productive conversation, respectful critique, questioning. Teacher prompts students to evaluate and justify arguments (theirs and others). Teacher probes student thinking to deepen understanding. Teacher provides rich, rigorous tasks that allow for deep analysis. Teacher effectively plans discourse, student groups, tasks. 	 Student provides examples and counterexamples. Student reasons inductively about data and context to construct argument. Student justifies the reasonableness of your or another's argument. Student uses a specific example to test a conjecture. Student supports arguments with mathematical evidence. Student asks clarifying questions. Student analyzes other's arguments.

Teachers	Students
 Teacher focuses on procedures and answers only. Teacher incorporates only low-level tasks or procedural practice. Teacher gives minimal feedback or feedback that only indicates correctness. Teacher maintains teacher directed conversations. Teacher does not allow time for discussion. 	 Student doesn't justify their reasoning. Student uses reasoning that is flawed (followed by no re-evaluation). Student applies mathematical definitions or assumptions incorrectly (followed by no re-evaluation). Student just gets the answer without considering what makes sense. Student doesn't engage in discourse.

MP4: Model with Mathematics

Definition

Mathematically proficient students can apply the math they know to solve problems in and out of school. They apply what they know and are comfortable making guesses or estimates to get started, realizing that these may need revised later. Students are able to identify important quantities in a practical situation and map their relationships using a variety of tools. They can analyze those relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense and make adjustments as needed.

Student Friendly

- I can use math to solve real world problems.
- I can use the appropriate strategy/model adjust my strategy/model if needed.
- I can make guesses or estimates to simplify a complicated situation, making revisions later.
- I can identify important quantities and relevant information.
- I can use an appropriate tool such as diagrams, tables, graphs, flowcharts and formulas.
- I can analyze my math to make conclusions.
- I can reflect on whether the results make sense.

Examples

Teachers	Students
 Teacher allows time for the process to take place. Teacher makes appropriate tools available. Teacher provides meaningful, real world, authentic and performance-based tasks and activities (non-traditional work problems). Teacher provides opportunities to investigate and integrate across curricula. 	 Student applies mathematics (numbers and symbols) to solve/work out real-life situations. Student shows evidence that they can use their mathematical results to determine if the results are reasonable. Student identifies important quantities in a practical situation. Student is comfortable making assumptions and approximations to simplify a complicated situation. Student analyzes relationships to draw conclusions.

Teachers	Students
 Teacher doesn't make tools available. Teacher doesn't allow time to process. Teacher provides only "traditional" work. Teacher doesn't allow time to investigate across curricula. Teacher uses "naked number" problems. 	 Student doesn't apply math to real world. Student does not show evidence of mathematical thinking. Student is not able to identify important quantities in a practical situation. Student is not comfortable making assumptions and approximations to simplify a complicated situation. Student does not draw conclusions.

<i>MP5: Use appropriate tools strategically</i>

Definition	Student Friendly
Students will recognize when and which tools	 I can use tools appropriately for
are needed to arrive successfully at a	solving problems and understand the
solution. These tools can provide	results.
connections and deeper understandings to	 I can determine the best tool to use
other mathematical concepts.	in a given situation.

Examples

Teachers	Students
 Teacher provides examples of when a tool can or should be used. Teacher models how to use tools. Teacher makes tools available in the classroom. Teacher encourages students to decide when a tool is needed through scaffolding. Teacher provides manipulatives to develop student understanding. 	 Student considers all available tools when solving a math problem and choose the best tool. Student uses a variety of tools to solve problems: paper pencil, protractor, calculator, manipulatives, computer Student practices using tools as needed or when first introduced. Student knows the tools available, and how to locate them.

Teachers	Students
 Teacher doesn't show students how to use tools. Teacher doesn't make tools available to students. Teacher doesn't give students a choice in using tools or require use of a specific tool. Teacher doesn't model thinking questions to encourage student reflection in deciding when to get a tool. 	 Student doesn't know where to find a tool or ask for a tool. Student doesn't attempt to use tools. Student doesn't know how to use tool(s). Student uses any tool available just because it is there.

MP6: Attend to Precision

Definition Mathematically proficient students communicate precisely. They use correct terminology in discussions and support with clear definitions in their own reasoning. Students state the meaning of the symbols they choose. They are careful about specifying units of measure, and labeling axes to clarify the relationship with quantities in a problem. They calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context.	 Student Friendly I can use appropriate mathematical vocabulary to explain my reasoning. I can use labels accurately. I can use symbols appropriately (ex. labeling axes, labeling answers with appropriate units, using the equal sign appropriately). I can calculate accurately and efficiently. I can use the most efficient strategy depending on the context.
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Examples

Teachers	Students
 Teacher models and encourages students to think aloud/talk aloud. Teacher creates a culture for guided inquiry where students work together and incorporate debriefing time for sharing and comparing strategies. Teacher promotes mathematical terminology/vocabulary. Teacher gives room to discuss mistakes, wrong answers and common misconceptions. Teacher values the process, not just the product. 	 Student communicates precisely - orally and written. Student uses mathematical concepts. Student uses symbols appropriately. Student uses vocabulary appropriately. Student attends to units/labeling/tools accurately. Student carefully formulates explanations. Student ensures reasonableness of answers. Student calculates accurately and efficiently.

Teachers	Students
 Teacher provides no time for discussion. Teacher doesn't allow students to work together. Teacher doesn't allow for sharing of strategies. Teacher doesn't expect appropriate vocabulary. Teacher only reviews answers with no discussion or explanation. 	 Student works in their head or on a calculator. Student doesn't use appropriate vocabulary. Student doesn't label answers with appropriate units. Student doesn't recognize if their answer makes sense.

MP 7 Look for and make use of structure

Definition Students routinely look for, interpret and use patterns or structures to model and solve problems. Students see complicated objects as single objects or being composed of several objects. Students make connections between strategies and problems previously solved to help solve new problems.	 Student Friendly I can use what I already know about math strategies and structure to solve a problem. I can identify patterns or structures and use them to make sense of a problem. I can step back to check my work and try a new strategy if needed.
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Examples

Teachers	Students
 Teacher helps students identify and evaluate efficient strategies for a solution. Teacher repeatedly breaks apart numbers and problems into different parts. Teacher models how to use what you know is true to solve a new problem. Teacher provides open-ended questions for student exploration. Teacher allows students time for discussion and processing. 	 Student recognizes, reflects on, and interprets patterns and structures. Student makes connections to skills and strategies previously learned to solve new problems and tasks. Student breaks down complex problems into simpler, more manageable chunks. Student shifts to different strategies when needed. Student looks for patterns. Student sees something as a whole or as a combination of parts.

Teachers	Students
 Teacher doesn't provide multiple methods and/or requires one method to solve a problem. Teacher isolates tasks/standards from one another; doesn't provide a connection for students. 	 Student looks at skills in isolation and doesn't applying previous content to new content. Student looks at a problem as a whole and not smaller parts.

MP8 Look for and express regularity in repeated reasoning.

Examples

Teachers	Students
 Teacher questions student solutions. Teacher promotes the connection between previously taught and new material. Teacher provides time for students to reflect on their work. Teacher provides opportunities that allow students to discover more efficient ways to problem solve. 	 Student continually checks back as they move forward. Student develop algorithms and shortcuts through repeated problem solving. Student keep track of the details without getting lost in the process.

Teachers	Students
 Teacher gives student an algorithm. Teacher focuses on basic/skilled based mathematical problems. Teacher does not allow for math discourse between students. Teacher isolates content. 	 Student continues to utilize concrete materials in order to arrive at a conclusion (i.e. using a number line to compute integers). Student doesn't connect details to the problem as a whole. Student does not reflect on results or connect prior learning to new learning.

FHSD Standards for Mathematical Practice Look-for Tool

The look-for tool was developed by the FHSD Course of Study Committee based off of the look-for tool created by the National Council of Supervisors of Mathematics (NCSM.)

"This look-for tool is a classroom resource that can be used as a non-evaluative assessment tool. The "look-fors" in this peer-observation tool are designed to assess the extent to which students are engaged in particular elements of the mathematical practices of the Common Core State Standards for mathematics. The tool provides suggested student responses for each of the mathematical practices as observational look-fors. These should be helpful in providing suggestions related to both planning and assessment for teachers, mathematical leaders and school based administrators.

The look-fors in this peer observation tool are designed to measure how well the instructor has integrated the essence of the principles of the Common Core for the mathematics into their classroom. There are eight key mathematical practices with specific look-fors for both student and teacher behaviors. Noting the occurrence and totality of these look-fors can be helpful in providing feedback and guidance for the instructor." NCSM

NOTE: All indicators are not necessary for providing full evidence of practice(s). Each practice may not be evident during every lesson.

Ма	thematics Practices	Student dispositions:	Teacher actions to engage students in Practices:
O v e r a r c h i	1. Make sense of problems and persevere in solving them	 Understand the problem Use strategies/strategic thinking to plan a solution pathway Use patience and persistence to solve problem Use self-evaluation and redirections Communicate both verbally and written Be able to deduce what is a reasonable solution Perseverance through multiple-step problems Comments: 	 Provide open-ended and rich problems Ask probing questions Model multiple problem-solving strategies Promotes and values discourse and collaboration Probe student responses (correct or incorrect) for understanding and multiple approaches Provide scaffolding appropriately Provide a safe environment for learning from mistakes Foster persistence/stamina in problem solving Comments:
n g h a b i t s	6. Attend to precision	 Communicate with precision-orally and written Use mathematics concepts, symbols and vocabulary appropriately Attend to units/labeling/tools accurately Carefully formulate explanations Ensure reasonableness of answers Calculate accurately and efficiently 	 Teacher models and encourages students to think aloud/talk aloud Guided Inquiry including teacher gives problem, students work together to solve problems, and debriefing time for sharing and comparing strategies Promote mathematical terminology/vocabulary Give room to discuss why wrong answers are wrong <u>Comments:</u>
R e a s o n	2. Reason abstractly and quantitatively	 Create and connect multiple representations (tables, pictures, words, symbols, graphs) Estimate first/answer reasonable De-contextualize Using context to think about a problem Attend to the meaning of the quantities, not just computing 	 Develop opportunities for and model problem solving strategies Tie content areas together to help make connections Give real world situations Value invented strategies and representations Less emphasis on the answer
, E x p I a i n	3. Construct viable arguments and critique the reasoning of others	 Ask questions Use examples and counterexamples (i.e., objects, diagrams, drawings, or actions) Reason inductively and make plausible arguments (<i>Does this always work?</i>) Analyze others arguments Comments: 	 Create a safe environment for risk-taking and critiquing with respect Use feedback to model desired student discourse Provide complex, rigorous tasks that foster deep thinking Plan effective questions, student grouping and time for student discourse Probe students Comments:

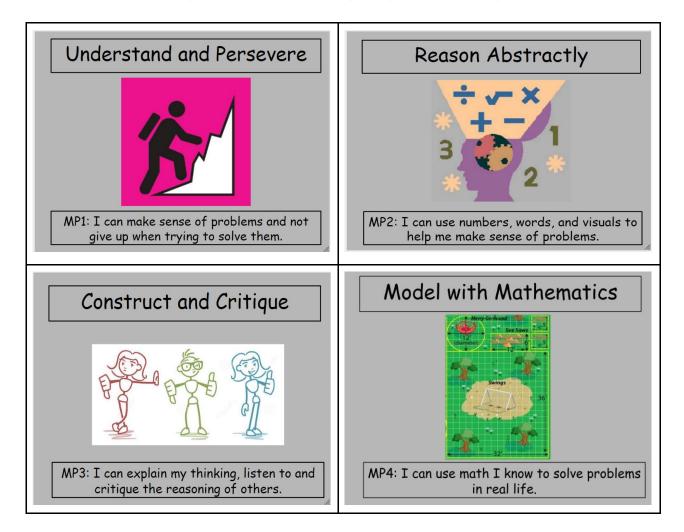
Math	nematics Practices	Student dispositions:	Teacher actions to engage students in Practices:
M o d i n g &	4. Model with Mathematics	 Apply mathematics (numbers and symbols) to solve/work out real-life situations Show evidence that they can use their mathematical results to think about a problem and determine if the results are reasonable Identify important quantities in a practical situation Be comfortable making assumptions and approximations to simplify a complicated situation Analyze relationships to draw conclusions 	 Allow time for the process to take place (model, make graphs, etc.) Make appropriate tools available Provide meaningful, real world, authentic and performance-based tasks (non- traditional work problems) Provide opportunities to investigate and integrate cross-curricularly Comments:
U s i g T o o I s	5. Use appropriate tools strategically	 Recognize the usefulness and limitations of different tools Choose the appropriate tool to solve a given problem and deepen their conceptual understanding (e.g., paper/pencil and manipulatives) Choose the appropriate technological tool to solve a given problem and deepen their conceptual understanding (e.g., spreadsheet, geometry software, calculator, web 2.0 tools) Comments 	 Maintain knowledge of appropriate tools Effective modeling of the tools available, their benefits and limitations Make available and encourage use of a variety of tools Comments:
G e n e r a I i	7. Look for and make use of structure	 Recognize, reflect on, and interpret patterns and structures Make connections to skills and strategies previously learned to solve new problems/tasks Use the structure of mathematics to solve simpler forms of the original problem Breakdown complex problems into simpler, more manageable chunks Be able to "step back" / shift perspective Comments: 	 Facilitate learning by using open-ended questioning to assist students in exploration Careful selection of tasks that allow for students to discern structures or patterns to make connections Be quiet and allow time for student discussion and processing in place of fixed rules or definitions Comments:
ı z n g	8. Look for and express regularity in repeated reasoning	 Maintain oversight of the process by continually evaluating the reasonableness of intermediate results Search for generalizations to identify and use short-cuts 	 Provide rich and varied tasks that allow students to generalize relationships and methods, and build on prior mathematical knowledge Provide adequate time for exploration, dialogue and reflection, peer collaboration Ask deliberate questions that enable students to reflect on their own thinking periodically throughout the process Create strategic and intentional check in points during student work time Comments:

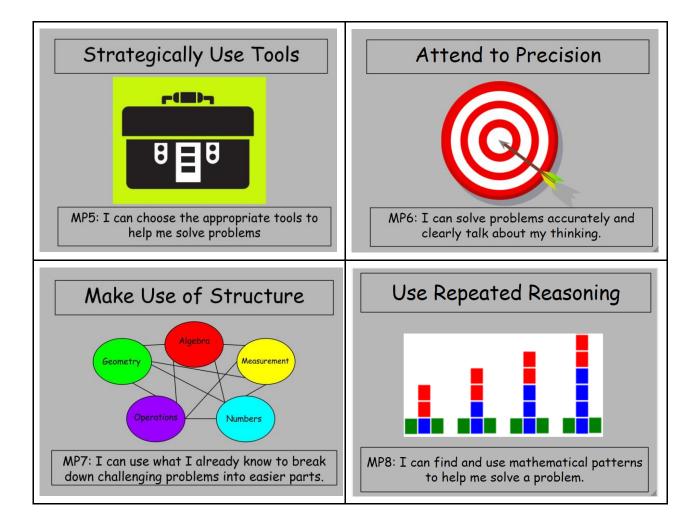
Mathematical Practice Classroom Visuals

The FHSD Course of Study Committee designed and approved the visuals. These visuals will be used in the Forest Hills classrooms as reminders to administrators, teachers, and students as to the importance of the practices. The practices tell us "how" the mathematics should look in the classroom.

Ideas of how to use the visuals

- display the posters in the classroom, reference them with students
- display student work next to the posters in the classroom
- use the visuals on assignments as a reminder for students of the expectations in the classroom
- post pictures of students "caught" in the act of one of the practices
- use the visuals on parent newsletters to acquaint parents with the practices





Section 3: FHSD Mathematical Teaching Habits (NCTM)

The teaching of mathematics is complex and requires teachers to have a deep understanding of the mathematical knowledge they are expected to teach. (Deborah Ball, NCTM) Teachers are expected to have a clear view of how student learning of that mathematics develops and progresses across grades. Also required of teachers is the need to be skilled at teaching in ways that effectively develop mathematics learning for all students. (Daro, Mosher, & Corcoran 2011)

Research conducted from cognitive science (National Research Council 2012) and mathematics education (Donovan and Bransford 2005) supports characterization of mathematics as an active process. Students build on his or her own mathematical knowledge from personal experiences, coupled with feedback from peers, teachers, and other adults, and themselves. The research has identified a number of principles of learning that provide the foundation for effective mathematics teaching.

The Eight Mathematics Teaching Practices (renamed, "Habits" by the FHSD course of study) provide a framework for strengthening the teaching and learning of mathematics. These habits represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics. The eight practices are based on the research mentioned above. Forest Hills Course of Study Committee has agreed that these practices should guide our professional learning opportunities for our teachers. (NCTM)

- 1. Establish mathematics goals to focus learning.
- 2. Implement tasks that promote reasoning and problem solving.
- 3. Use and connect mathematical representations.
- 4. Facilitate meaningful mathematical discourse.
- 5. Pose purposeful questions.
- 6. Build procedural fluency from conceptual understanding.
- 7. Support productive struggle in learning mathematics.
- 8. Elicit and use evidence of student thinking.

1. Establish Mathematics Goals to Focus Learning

Effective Teaching of Mathematics

- establishes clear goals for the mathematics that students are learning
- situates goals within learning progressions
- uses the goals to guide instructional decisions
- focuses student attention on monitoring their own progress toward the learning outcomes

Teacher Actions	Student Actions
Establish clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit.	Engage in discussions of the mathematical purpose and goals related to their current work in the mathematical classroom (What are we learning? Why are we learning it?)
Identify how goals fit within a mathematics learning progression.	Use the learning goals to stay focused on their progress in improving their understanding of mathematics content and proficiency in using
Discuss and refer to the mathematical purpose and goal of a lesson during instruction to ensure	mathematical practices.
that students understand how the current work contributes to their learning.	Connect their current work with the mathematics that they studied previously and see where the mathematics is going.
Use the mathematics goals to guide lesson planning and reflection and to make in-the- moment decisions during instruction.	Assess and monitor toward the mathematics learning goals.

Instructional Research Based Strategies

- "formulating clear, explicit learning goals sets the stage for everything else" (Heibert)
- "goals should be situated within mathematical learning progressions and connected to the big ideas" (Daro, Mosher and Corcoran)
- "classrooms where students understand the learning expectations for their work perform at higher levels than those classrooms where it is unclear" (Marzano, Haystead, Hattie)
- "student friendly goals can be discussed within a lesson so that students see value in and understand the purpose of their work" (Black and William, Marzano)
- "as teachers establish specific goals and consider the connections to broader math topics, they become more
 prepared to use goals to make decisions during instruction" (Hiebert)

Important Instructional Considerations

- goals communicate what students will understand based on standards
- goals clarify and help teachers understand the mathematical expectations for student learning.
- goals identify mathematical practices that students are learning to use more proficiently
- goals should not just be reiteration of standards but should be linked to curriculum and student learning
- goals should be situated within the mathematical landscape to support opportunities to build connections so that students see how ideas build
- mathematical purpose of the lesson should not be a mystery to students

Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them.
- 3. Construct viable arguments and critique the reasoning of others.
- 6. Attend to precision.

Instructional Materials Statement

FHSD has adopted *Investigations, Edition 3*, as the Tier 1 main resource for Kindergarten. Use of this resource and other standards aligned resources should be used as advised in the FHSD curriculum maps. The learning target statements in the FHSD curriculum maps state what students should know and be able to do based on the Ohio Learning Standards.

FHSD has adopted Eureka Math as the Tier 1 main resource for grades 1-6. Use of this resource, Investigations and

other standards aligned resources should be used as advised in the FHSD curriculum maps. The learning target statements in the FHSD curriculum maps state what students should know and be able to do based on the Ohio Learning Standards.

FHSD has adopted *Big Ideas* as the Tier 1 resource for grades 7-8. Use of this resource, *Connected Math Project* and other standards aligned resources should be used as advised in the FHSD curriculum maps. The learning target statements in the FHSD curriculum maps state what students should know and be able to do based on the Ohio Learning Standards.

Student Materials

FHSD Learning Targets FHSD Standards of Mathematical Practice Posters

Instructional Resources

District created/purchased FHSD Elementary Curriculum Maps Grades 7-8 Curriculum Maps Classroom Assessment for Student Learning, Doing It Right-Using It Well, Jan Chappuis and Rick Stiggins TenMarks-district purchased grades 2-8 YummyMath-district purchased grades 7 & 8 Mathalicious-district purchased grades 7 & 8

Additional resources

2. Implement tasks that promote reasoning and problem solving

Effective teaching of mathematics

- engages students in solving and discussing tasks that promote mathematical reasoning and problem solving
- allows for multiple entry points and varied solution strategies

Teacher Actions	Student Actions
Motivate students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding. Select tasks that provide multiple entry points through the use of varied tools and representations. Pose tasks on a regular basis that require a high level of cognitive demand. Support students in exploring tasks without taking over student thinking. Encourage students to use varied approaches and strategies to make sense of and solve tasks.	Persevere in exploring and reasoning through tasks. Take responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas. Use tools and representations as needed to support their thinking and problem solving. Accept and expect that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another.

Instructional Research Based Strategies

- "...mathematical tasks are viewed as placing *higher-level cognitive demands* on students when they allow students to engage in active inquiry and exploration or encourage students to use procedures in ways that are meaningfully connected with concepts or understanding." (Smith and Stein, 1998) Classrooms where high-level thinking and reasoning occurs shows the greatest student learning.
- "Tasks that encourage students to use procedures, formulas, or algorithms in ways that are not actively linked to meaning, or that consist primarily of memorization or the reproduction of previously memorized facts, are viewed as placing *lower-level cognitive demands* on students." (Smith and Stein, 1998)
- A task should provide students with the opportunity to engage actively in reasoning, sense making, and problem solving so that they develop a deep understanding of mathematics (NCTM)

Important Instructional Considerations

- Provide rich, open ended tasks that allow for multiple solutions
- Rich open ended task that promote reasoning and problem solving are in context and take an entire class period
 or more.
- Model and use visual supports, explore patterns
- Prior knowledge must be considered when considering the level of the cognitive demand.
- Consider the four "Levels of Cognitive Demand" from Smith and Stin
- Use Webb's Depths of Knowledge to guide high-level of task selection
- Discussion strategies such as Harkness Method
- Use of instructional strategy of, "Launch, Explore, Summarize"
- Must engage on a regular basis in high level tasks to learn mathematics with understanding.

Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them.
- 3. Construct viable arguments and critique the reasoning of others.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.

Instructional Materials Statement

FHSD has adopted *Investigations, Edition 3*, as the Tier 1 main resource for Kindergarten. Use of this resource and other standards aligned resources should be used as advised in the FHSD curriculum maps. The learning target statements in the FHSD curriculum maps state what students should know and be able to do based on the Ohio Learning Standards.

FHSD has adopted *Eureka Math* as the Tier 1 main resource for grades 1-6. Use of this resource, Investigations and other standards aligned resources should be used as advised in the FHSD curriculum maps. The learning target statements in the FHSD curriculum maps state what students should know and be able to do based on the Ohio Learning Standards.

FHSD has adopted *Big Ideas* as the Tier 1 resource for grades 7-8. Use of this resource, *Connected Math Project* and other standards aligned resources should be used as advised in the FHSD curriculum maps. The learning target statements in the FHSD curriculum maps state what students should know and be able to do based on the Ohio Learning Standards.

Student Materials

Kindergarten Investigations, edition 3-Student Edition Grades 1-6 Eureka Math-student edition Grades 7 & 8 Big Ideas Grades 2-8 TenMarks Math

Students should have access to a variety of mathematical tools to use while solving problems and representing their thinking. Materials may include, but not are limited to the list below.

base ten blocks measurement tools pattern blocks graph paper geoboards Cuisenaire rods algebra tiles tens frames 1in. or cm tiles calculator tangrams rekenreks connecting cubes 100s charts number lines diagrams two color counters fractions pieces

Instructional Resources District created/purchased

 FHSD Curriculum Maps

 Grades 7-8 Curriculum Maps

 Kindergarten-Investigations, edition 3

 Grades 1-6 Eureka Math

 Grades 7-8 Big Ideas

 Number Talks: Whole Number Computation, Grades K–5

 It Makes Sense! Using the Hundreds Chart to Build Number Sense, Grades K-2 First Edition

 It Makes Sense! Using the Ten Frames to Build Number Sense, Grades K-2 First Edition

 TenMarks-district purchased grades 2-8

 YummyMath-district purchased grades 7 & 8

 Mathalicious-district purchased grades 7 & 8

Additional resources

NCTM Navigations Series <u>NCTM Illuminations</u> <u>Illustrative Math</u> Lessons for Algebraic Thinking (Lawrence & Hennessy) Achieve the Core

3. Use and Connect Mathematical Representations

Effective teaching of mathematics

 engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tool for problem solving

Teacher Actions	Actions of Student
Select tasks that allow students to decide which representations to use in making sense of the problems.	Use and explore multiple forms of representation to make sense of and understand mathematics.
Allocate substantial instructional time for students to use, discuss, and make connections among representations.	Describe and justify mathematical understanding and reasoning using effective representations / drawings, diagrams, words,
Introduce forms of representations that can be useful to students.	Make choices about which forms or representations to use as tools for solving problems.
Ask students to make math drawings or use other visual supports to explain and justify their	Make sense of problems through use of tables, drawings, diagrams and other representations.
reasoning.	Connect mathematical ideas and concepts to real- world situations and contexts.
Focus students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation.	Consider advantages or suitability of using alternate representations when problem solving.
Design ways to elicit and assess students' abilities to use representations meaningfully to solve problems.	

Instructional Research Based Strategies

- When students learn to represent, discuss, and make connections among mathematical ideas in multiple forms, they demonstrate a deeper understanding and enhanced problem-solving abilities. (Fuson, Kalchman, & Bransford 2005; Lesh, Post and Behr 1987)
- Connections should be made among representations which include; visual, contextual, verbal, physical and symbolic. The multiple lenses of representation make the perspective of the picture deeper and richer.
- The depth of understanding is related to the strength of connections among mathematical representations that students have internalized. (Pape and Tchoshanov 2001; Webb, Boswinkel and Dekker 2008)
- Visual representations are of particular importance in the mathematics classroom, helping students to advance their understanding of mathematical concepts and procedures, make sense of problems and engage in mathematical discourse. (Arcavi 2003; Stylianou and Silver 2004)
- Success in solving problems is also related to students' ability to move flexibly among representations. (Huinker 2013; Stylianou and Silver 2004)

Important Instructional Considerations

- Math drawings and other visual supports are of particular importance for English language learners, learners with special needs, or struggling learners, because they allow more students to participate meaningfully in the mathematical discourse in the classroom (Fuson and Murata 2007).
- Help students to see the connection between the different representations. It isn't just about showing multiple ways but seeing that the representations are connected to each other mathematically.

Standards for Mathematical Practice

- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.

Instructional Materials Statement

FHSD has adopted *Investigations, Edition 3*, as the Tier 1 main resource for Kindergarten. Use of this resource and other

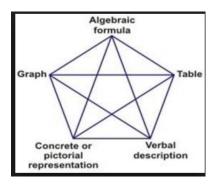
standards aligned resources should be used as advised in the FHSD curriculum maps. The learning target statements in the FHSD curriculum maps state what students should know and be able to do based on the Ohio Learning Standards.

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Student Materials

Multiple representations include:



A variety of tools/manipulatives should be available to students to promote understanding and sense making of the mathematics. (This is not a limited list, but suggestions of tools to have available.)

double number line

connecting cubes

two color counters

100s charts

diagrams

number lines

fraction pieces

number bonds base ten blocks measurement tools pattern blocks graph paper geoboards Cuisenaire rods

- tape diagram algebra tiles tens frames 1 in. or cm tiles calculator tangrams rekenreks
- Instructional Resources

FHSD Curriculum Maps Grades 7-8 Curriculum Maps Kindergarten-Investigations, edition 3 Grades 1-6 Eureka Math Grades 7-8 Big Ideas Number Talks: Whole Number Computation, Grades K-5 It Makes Sense! Using the Hundreds Chart to Build Number Sense, Grades K-2 First Edition It Makes Sense! Using the Ten Frames to Build Number Sense, Grades K-2 First Edition TenMarks-district purchased grades 2-8 YummyMath-district purchased grades 7 & 8 Mathalicious-district purchased grades 7 & 8 Additional resources http://map.mathshell.org/materials/index.php https://www.engageny.org/ https://investigations.terc.edu/ https://www.illustrativemathematics.org/ http://mathsolutions.com/ **NCTM Navigations Series NCTM Illuminations Illustrative Math** Lessons for Algebraic Thinking (Lawrence & Hennessy) Achieve the Core

4. Facilitate meaningful mathematical discourse.

Effective teaching of mathematics

• facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments

Teacher Actions	Student Actions
Engage students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.	Present and explain ideas, reasoning, and representations to one another in pairs, small-group, and whole-class discourse.
Scaffold student approaches and solution strategies for whole-class analysis and discussion.	Listen carefully to and critique the reasoning of peers, use examples to support or counterexamples to refute arguments.
Facilitate discourse among students by having them justify and explain their reasoning for their answer and approach.	Seek to understand the approaches used by peers by asking clarifying questions, try out others' strategies, and describe the approaches used by others.
Ensure progress toward mathematical goals by making explicit connections to student approaches and reasoning.	Identify how approaches to solving a task are the same and how they are different.

Instructional Research Based Strategies

- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics. (Michaels, O'Conner and Resnick)
- Students who learn to articulate and justify their own mathematical ideas, reason through their own and others mathematical explanations, and provide a rationale for their answers develop a deep understanding that is critical to their future success in mathematics and related fields. (Carpenter, Franke, and Levi)
- Whole class discussions strategies to support discourse
 - anticipate student responses prior to the lesson
 - o monitor students work and engagement with the tasks
 - select particular students' responses in a specific order to promote a variety of strategies for conceptual understanding
 - sequencing student responses to scaffold the learning
 - o connect different students' responses to key mathematical ideas
- Mathematical discourse includes the purposeful exchange of ideas through classroom discussion as well as through other forms of verbal, visual, and written communication.
- Using discourse gives students opportunities to share ideas and clarify understandings, construct convincing
 arguments regarding why and how things work, develop a language for expressing mathematical ideas, and
 learn to see things from other perspectives.
- Teachers and students proceed through the levels in shifting from a classroom in which teachers play the leading
 role to one where they facilitate students' mathematical thinking.

Important Instructional Considerations

- Teacher Role- Students carry the conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for the students to clarify thinking of others.
- Questioning- Student to student talk is student initiated. Students ask questions and listen to responses.
- Questions include asking, "why?" and calling for justification. Teacher questions may still guide discourse.
 Explaining Mathematical Reasoning- Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher.
- Mathematical Representations- Students follow and help shape the descriptions of others math thinking through math representations and may suggest edits in other's representation.
- Building student responsibility within the community- Students believe that they are math leaders and can help shape the thinking of others. They help shape others' math thinking in supportive, collegial ways and accept the same support from others.

Standards for Mathematical Practices

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with Mathematics.

- 6. Attend to precision.
- 8. Look for and express regularity in repeated reasoning.

Instructional Materials Statement

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Student Materials

Materials will be dependent on the lesson; they will depend on representations, communication tools, justifying, and responsive classroom techniques. These could include but are not limited to: journals, online collaborative tools

Instructional Resources

District created/purchased FHSD Curriculum Maps Grades 7-8 Curriculum Maps Kindergarten-Investigations, edition 3 Grades 1-6 Eureka Math Grades 7-8 Big Ideas Number Talks: Whole Number Computation, Grades K–5 Number Talks-Sherry Parrish It Makes Sense! Using the Hundreds Chart to Build Number Sense, Grades K-2 First Edition It Makes Sense! Using the Ten Frames to Build Number Sense, Grades K-2 First Edition It Makes Sense! Using the Ten Frames to Build Number Sense, Grades K-2 First Edition TenMarks-district purchased grades 2-8 YummyMath-district purchased grades 7 & 8 Mathalicious-district purchased grades 7 & 8

Additional resources

Generating Math Talk- Marilyn Burns Classroom Discussions using Math Talk - Marilyn Burns The First Six Weeks of School-Paula Denton NCTM Illuminations Illustrative Math Achieve the Core

Recommended Reading List

Classroom Discussions: Using Math Talk to Help Students Learn, Grades K-6, Second Edition, S. H. Chapin, C. O'Connor, and N.C. Anderson.

Classroom Discussions: Seeing Math Discourse in Action, Grades K-6, N.C. Anderson, S.H. Chapin, C. O'Connor, (Copyright © 2011 by Scholastic, Inc.)

5. Pose purposeful questions

Effective teaching of mathematics

 Uses purposeful questions to assess and advance students' ability to reason and make sense of important mathematical ideas & relationships

Teacher Actions	Student Actions
Advance student understanding by asking questions that build on student thinking, but do not take over or funnel ideas to lead students to a desired conclusion.	Expect to be asked to explain, clarify, and elaborate on their thinking verbally and in written format. Think carefully about how to present their
Ask questions that go beyond gathering information. The questions should probe students' thinking and require explanation and justification.	Reflect on and justify their reasoning, not simply provide answers.
Ask intentional questions that make the mathematics more visible and accessible for student examination and discussion.	Listen to, comment on, and question the contributions of their classmates.
Allow sufficient wait time so that more students can formulate and offer responses.	

Instructional Research Based Strategies

- Depth of Knowledge Question Stems (Webb, Norman)
- Depth of Knowledge
- Focusing Pattern of Questioning (Wood, Terry)
- <u>http://www.svmimac.org/images/SVMIPD.091312.Questioning_our_Patterns.pdf</u>
- Rigor Relevence Framework (Daggett, William) http://www.leadered.com/pdf/rigor_relevance_framework_2014.pdf

A Framework for types of questions used in mathematics teaching (e.g. Boaler and Brodie 2004; Chapin and O'Connor 2007)

1. <u>Gathering Information</u> - Students recall facts, definitions, or procedures. ex. When you write an equation, what does that equal sign tell you? What is the formula for finding the area of a rectangle? What does the interquartile range indicate for a set of data?

2. <u>Probing Thinking</u> - Students explain, elaborate, or clarify their thinking including articulating the steps in solution methods or the completion of a task. ex. As you drew that number line, what decisions did you make so that you could represent 7/4 on it? Can you show and explain more about how you used a table to find the answer to the Smartphone Plans tasks? It is still not clear how you figured out that 20 was the scale factor, so can you explain it another way?

3. <u>Making the Mathematics Visible</u> - Students discuss mathematical structures and make connections among mathematical ideas and relationships. ex. What does your equation have to do with the band concert situation? How does that array relate to multiplication and division? In what ways might the normal distribution apply to this situation?

4. <u>Encouraging Reflection and Justification</u> - Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work. ex. How might you prove that 51 is the solution? How do you know that the sum of two odd numbers will always be even? Why does plan A in the Smartphone Plans task start out cheaper but becomes more expensive in the long run?

Important Instructional Considerations

- Teacher plans questions that will be asked.
- Questioning should encourage students to explain and reflect on their thinking and reasoning; leading them to a
 desired procedure or conclusion.
- Questioning should allow teachers to discern what students know and adapt lessons to meet varied levels of understanding, help students make important mathematical connections and support students in posing their own questions.

- Questions should attend to what students are thinking, pressing them to communicate their thoughts clearly, and
 expecting them to reflect on their thoughts and those of their classmates.
- Questioning should provoke connections to prior learning.
- Questioning should promote use of appropriate vocabulary.
- Use two types of questioning strategies for the teacher; funneling-a set of questions to lead students to a desired
 procedure or conclusion. Focusing-listening to what the student is thinking, pressing them to communicate
 clearly and expecting the student to reflect on their thoughts and their peer's thoughts.

Standards for Mathematical Practice

- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

Instructional Materials Statement

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FHSD has adopted *Big Ideas* as the Tier 1 resource for grades 7-8. Use of this resource, *Connected Math Project* and other standards aligned resources should be used as advised in the FHSD curriculum maps. The learning target statements in the FHSD curriculum maps state what students should know and be able to do based on the Ohio Learning Standards.

Student Materials

Investigations student edition *Eureka* student edition Journals

Instructional Resources

District created/purchased FHSD Curriculum Maps Grades 7-8 Curriculum Maps Kindergarten-Investigations, edition 3 Grades 1-6 Eureka Math Grades 7-8 Big Ideas Number Talks: Whole Number Computation, Grades K–5 Number Talks-Sherry Parrish DOK question stems

Additional resources

Generating Math Talk- Marilyn Burns Classroom Discussions using Math Talk - Marilyn Burns The First Six Weeks of School-Paula Denton NCTM Illuminations Illustrative Math Achieve the Core Good Questions for Math Teaching: Why Ask Them and What to Ask, K-6 Good Questions for Math Teaching: Why Ask Them and What to Ask, Grades 5-8 Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K-5 Extending Children's Mathematics: Fractions & Decimals: Innovations In Cognitively Guided Instruction Intentional Talk: How to Structure and Lead Productive Mathematical Discussions Question Stems to promote 8 mathematical practices Quick Guide to Questioning in the Classroom

6. Build procedural fluency from conceptual understanding.

Effective teaching of mathematics promotes students to be able to

- Build fluency with procedures on a foundation of conceptual understanding
- Become skillful in flexibly using procedures as they solve contextual and mathematical problems

Teacher Actions	Student Actions
Acknowledge the importance of both conceptual understanding and procedural fluency but also ensure that the learning of procedures is developed over time, on a strong foundation of understanding and the use of student-generated strategies in solving problems. Provide students opportunities to use their own reasoning strategies and methods for solving problems. Ask students to discuss and explain why the procedures that they are using work to solve particular problems. Connect student generated strategies and methods to more efficient procedures as appropriate. Use visual models to support students' understanding of general methods. Provide students with opportunities for continuous practice of procedures. Provide students time to practice math facts.	Demonstrate the ability to choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and are able to explain their approaches, and they are able to produce accurate answers efficiently. Demonstrate knowledge by practicing on a moderate number of carefully selected problems once they have a strong conceptual foundation and can explain the use of the strategy. Access procedures that they can use with understanding on a broad range of problems. Know which procedure is appropriate and most productive in a given situation.
 Provide students opportunities to use their own reasoning strategies and methods for solving problems. Ask students to discuss and explain why the procedures that they are using work to solve particular problems. Connect student generated strategies and methods to more efficient procedures as appropriate. Use visual models to support students' understanding of general methods. Provide students with opportunities for 	moderate number of carefully selected proble once they have a strong conceptual foundation and can explain the use of the strategy. Access procedures that they can use with understanding on a broad range of problems. Know which procedure is appropriate and mo

Instructional Research Based Strategies

"Strategies that Work" Marzano:

Non-linguistic Representations: Ask students to...

- Generate mental images representing content
- Draw pictures or pictographs representing content
- Construct graphic organizers representing content
- Act out content
- Make physical models of content
- Make revisions in their mental images, pictures, pictographs, graphic organizers, and physical models

Important Instructional Considerations

Standards for Mathematical Practice

- 4. Model with mathematics
- 6. Attend to precision
- 7. Look for and make use of structure
- 8. Look for and express regularity in repeated reasoning

Instructional Materials Statement

Math Rep District Suggestions for Achieving Procedural Fluency from Conceptual Understanding

After researching procedural fluency, fact fluency, best practice for learning addition/subtraction and multiplication/division facts and other operational procedures, the 2015-2016 FHSD Math Reps created a document to help the teachers of Forest Hills use best practice to move students from conceptual understanding to fluency with facts and procedures as

identified in Ohio's Learning Standards.

The document linked after this paragraph, uses DuFour's four questions to identify at each grade level what is expected to be learned, how we should check to see if what they have learned and then suggestions of what to do if they don't have fluency and what to do if they have fluency based on the assessment suggestions.

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Student Materials

A variety of tools/manipulatives should be available to students to promote understanding and sense making of the mathematics. (This is not a limited list, but suggestions of tools to have available.)

number bonds base ten blocks measurement tools pattern blocks graph paper geoboards Cuisenaire rods tape diagram algebra tiles tens frames 1 in. or cm tiles calculator tangrams rekenreks double number line connecting cubes 100s charts number lines diagrams two color counters fraction pieces

Instructional Resources

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Additional resources

See FHSD guidelines for suggestions on procedural and fact fluencies

7. Support productive struggle in learning mathematics

Effective teaching of mathematics

- Consistently provides students with opportunities and supports to engage in productive struggle
- Opportunities for delving more deeply into understanding the mathematical ideas
- Able to apply their learning to new problem situations

Teacher Actions	Student Actions
Anticipate what students might struggle with during a lesson and being prepared to support them productively through the struggle.	Struggle at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle.
Give students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them.	Ask questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks.
Help students realize that confusion and errors are a natural part of learning, by facilitating discussion on mistakes, misconceptions, and struggles. Praise students for their efforts in making sense	Persevere in solving problems and realizing that it is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up. Help one another without telling their classmates what the answer is or how to solve the problem.
of mathematical ideas and perseverance in reasoning through problems.	

Instructional Research Based Strategies

- "Rescuing" students when they face difficulties undermines the efforts of students, lowers the cognitive demands of the task, and deprives the students of opportunities to engage fully in making sense of the mathematics. (Reinhart 2000; Stein et al. 2009)
- "If you are not struggling, you are not learning". (Carter 2008, p.136)
- Provide students with specific descriptive feedback on their progress related to their making sense of math. (Clarke 2003; Hattie and Timperley 2007)
- <u>Mathematical Mindset</u>, preview chapters from Jo Boaler
- <u>Constructive Struggle, The Value of Challenging Our Students</u> by Cathy Seeley

Important Instructional Considerations

- Create a safe environment for learning.
- Consider and address student struggles and misconceptions.
- Consider and discuss growth mindset and the importance of mistakes for learning.
- Classrooms should embrace productive struggle to necessitate rethinking on the part of both the teacher and student.
- Students question and critique the reasoning of their peers and reflect on their own understanding in an appropriate manner.
- Students have access to tools that will support their thinking processes.
- Teacher plans for tasks that promote reasoning and problem solving; a solution pathway is not straightforward, but requires some struggle to arrive at the solution.
- Model productive struggle and how to generate conversations between students.

Standards for Mathematical Practices:

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 6. Attend to precision.

Instructional Materials Statement

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Student Materials:

variety of manipulatives paper-pencil access to technology; calculator models of multiple strategies for number operations access to formulas steps to problem solving poster/guide graphic organizers math journals for reflection/discussion

Instructional Strategies/Resources:

variety of rich problems real world task problems related to unit models launch, explore, summarize think-pair-share setting up problem-solving expectations/guide small group work - establish roles for each group member concept mapping Lit Circles Harkness Method

District created/purchased

 FHSD Curriculum Maps

 Grades 7-8 Curriculum Maps

 Kindergarten-Investigations, edition 3

 Grades 1-6 Eureka Math

 Grades 7-8 Big Ideas

 TenMarks-district purchased grades 2-8

 YummyMath-district purchased grades 7 & 8

 Mathalicious-district purchased grades 7 & 8

Additional resources

Triumph Learning-Productive struggle white paper Emergentmath.com -The struggle for productive struggle NCTM Navigations Series NCTM Illuminations Illustrative Math

8. Elicit and use evidence of student thinking

Effective teaching of mathematics

- Uses evidence of student thinking to assess progress toward understanding
- Uses evidence to adjust instruction continually in ways that support and extend learning

Teacher Actions	Student Actions
Identify what counts as evidence of student progress toward mathematics learning goals. (Formative Assessment)	Reveal mathematical understanding, reasoning, and methods in written work and classroom discourse.
Elicit and gather evidence of student understanding at strategic points during instruction.	Reflect on mistakes and misconceptions to improve their mathematical understanding. Ask questions, respond to, and give suggestions to support the learning of their classmates. Assess and monitor their own progress toward mathematics learning goals and identifying areas in which they need to improve.
Interpret student thinking to assess mathematical understanding, reasoning and methods.	
Make in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend.	
Reflect on evidence of student learning to inform the planning of next instructional steps.	

Instructional Research Based Strategies

- Attention to eliciting and using evidence is an essential component of formative assessment. (William 2007a)
- "Teachers using assessment for learning continually look for ways in which they can generate evidence of student learning, use this evidence to adapt their instruction to better meet their students' learning needs." (Leahy, 2005)
- "Identifying indicators of what is important to notice in students' mathematical thinking, planning for ways to elicit that information, interpreting what the evidence means with respect to students' learning, and then deciding how to respond on the basis of students' understanding." (Jacobs, Lamb, & Philipp 2010: Sleep and Boerst 2010; van Es 2010)

Important Instructional Considerations

- What mathematical domain is being taught
- Developmental considerations
- "Teachers attend to more than just whether an answer is or is not correct." (Crespo 2000)
- Each lesson needs to include intentional and systematic plans to elicit evidence that will provide information about how student learning is evolving toward the desired goal. (Heritage 2008, p. 6)

Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

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Student Materials

Opportunities:

- to say, draw, build, write (paper/pencil, manipulatives) to show thinking (including difficulties, mistakes, misconceptions)
- discussion time
- for students to make and use connections among mathematical representations

Instructional Resources District created/purchased

FHSD Curriculum Maps Grades 7-8 Curriculum Maps Kindergarten-Investigations, edition 3 Grades 1-6 Eureka Math Grades 7-8 Big Ideas TenMarks-district purchased grades 2-8 YummyMath-district purchased grades 7 & 8 Mathalicious-district purchased grades 7 & 8

Assessment resources

- assessments (pre, post, informal/formative, summative, district created unit assessments)
- daily check-ins (exit slips, sprints, homework, fluency practice, district created formative assessments)
- high-level tasks that require students to explain, represent, and justify mathematical understanding and skills (promoting reasoning and problem solving)

Additional resources

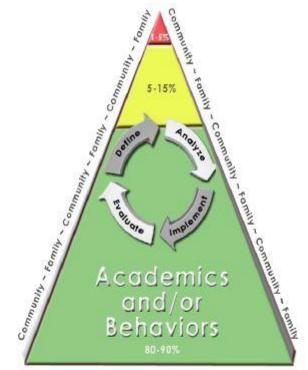
Howard County Assessments

Section 4: Response to Intervention

The Forest Hills Way...

We believe all students can learn. We also believe that to truly be a high achieving district, we must meet the varying needs of our students from support instruction to rigorous coursework to enrichment opportunities. By using a systematic approach to know our students as learners including data analysis and Response to Intervention we maximize the learning potential for individual students. RTI provides a framework for this.

A multi-tiered system of support is put in place, including strong core curriculum and differentiated instruction at a Tier 1 (classroom) level. Response to Intervention (RTI) includes screening of all of our students, and utilization of data to guide educational decisions in a problem solving model. Much as doctors use screens such as temperature and blood pressure checks to help determine which patients may require a deeper diagnosis with possible treatment and monitoring, we use a universal screener, to help identify students who need further attention and more careful consideration. The purpose of screening activities is to predict possible academic and behavior problems as well as to identify areas of student strength. Students demonstrating needs, based on data based decision rules will receive instruction and proven, research based intervention at varying levels of intensity. By monitoring student achievement throughout intervention, we are able to adjust the level of needed support.



Community ~ Family ~ Community ~ Family ~ Community ~ Family

Rtl One Page- Overview

Tier 1	Tier 2	Tier 3
Research-based core curriculum and instructional strategies	Targeted instruction	Intensified targeted support
 Should be: Core Curriculum which all students access Research-based instructional strategies such as Marzano's Key Strategies Visible Learning Key Strategies Explicit Instruction (I do, We do, You do) Differentiated Enriched for higher-performing students Scaffolded and supported for o English Language Learners o Students With Disabilities 	 Should be: Small group Supplemental to tier 1 Targeted instruction focused on specific skills and/or learning strategies Frequently progress monitored Explicit Timely (starting efficiently and lasting at least 4-6 weeks) Planned Documented 	 Should be: Determined through collaborative problem solving Inclusive of parents, teachers, and all other relevant people A smaller teacher-student ratio More intensive supplemental interventions (frequency, duration, group size) More frequently progress monitored Comprehensive (addressing all areas of concern) The "go to" process when Tier 2 is not enough

<u>FHSD Math Flowchart</u>- The flowchart is the guidance document for making math instructional decisions based on student assessment data. Assessment resources and instructional tools aligned to mathematical needs and supports are listed; accompanied by a description for use.

Subskills and Scaffolds to support grade level standards

The subskills are based on the Ohio Learning Standards and FHSD learning targets found in the FHSD curriculum maps.

Mathematics and the Gifted Student

Forest Hills School District provides a "Math Plus" class to students in grades 4, 5, and 6. This daily advanced math class includes interdisciplinary units connecting math curriculum to Language Arts, Social Studies and Science content. This class is designed for students identified as *Superior Cognitive* on a state approved standardized intelligence test **OR** with a cognitive ability of 122 or above and a Math Achievement composite score of 97th percentile or above on the most recently administered nationally normed tests that are on the state approved list.

Ohio's New Learning Standards provide the foundation for the curriculum within the "Math Plus" class along with additional supplemental materials that challenge high cognitive or high math ability students. Differentiation for these students is critical due to their vastly different academic and social needs.

Instruction within the Math Plus classroom combines rigorous problem solving strategies with compacted daily instruction in math. This instruction includes extensions allowing students to explore areas of interest and talent. Students explore mathematical processes through inquiry to become innovators of mathematical applications. The instruction provides opportunities for students to learn beyond the curriculum in problem-based learning applying their knowledge to real world problems. Students are guided as they explore their passions in 21st century learning. Students also are challenged to discover more about themselves as the social emotional needs of gifted students are nurtured.